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Concrete-Representational-Abstract and Multisensory Strategies: An Inclusive Approach to Mathematics

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Abstract

Maths is an area of processing which many students in school continue to struggle with, whether or not they have learning difficulties of any kind. In this article, the authors review the usefulness of an inclusive approach to Maths using the CRA, Concrete-Representational-Abstract, which builds on multisensory approach suitable for all learners. Outlining the Piagetian principles first underlying learning, the authors demonstrate in practical terms how this system is ideally delivered, providing supportive evidence for the impact of the approach in students with learning difficulties. The role of the skilled teacher in ensuring this is delivered effectively joining together the 3 sections explicitly to aid understanding, is emphasized here.

Keywords: Concrete-Representational-Abstract (CRA), Maths, Learning difficulties

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INTRODUCTION.

Mathematics, for some reason, comes across as a challenging subject for the majority of the students, particularly those with learning differences (Baker, Gersten and Dae-Sik, 2002). According to Piaget's Four Stages of Mental Development, children around the ages of 7 to 11 years are said to be in the Concrete Operational Stage, a stage where the development of ideas and thinking tend to be closely tied to performing actions on physical objects. This is especially so as the concepts and instructional methods become more abstract.

The National Council of Teachers of Mathematics makes it known that all students benefit from the use of manipulatives and visual aids (Shaw, 2002). "A major responsibility of teachers is to create a learning environment in which students' use of multiple representations is encouraged, supported, and accepted by peers and adults." (NCTM, 2000, p. 139). Sadly, in our schools in India, concrete experiences are limited to pre-primary education.

When we solve mathematical problems, a core part of the solution process is how we represent the ideas in the problem. The form of representation we select allows us to manipulate the information (as opposed to manipulating symbols) to reach a sensible solution. With the Representation Standard, our goal is for young students to show their mathematical ideas and procedures in multiple ways. Through the use of representations, children develop their own mental images of mathematical ideas. Representations used in the early grades may not be those that are traditionally used by adults. Students' representations provide a record of their efforts to understand mathematics and to make that understanding accessible to others. It is through representations that we can get into a child's thinking, assess the child's understanding, and make instructional decisions.

Students in the early grades would benefit from using multiple modes to represent their thinking. Because of the Concrete Operational Stage that Piaget suggested they are in, most of them would be comfortable with moving physical objects to show their thinking about a problem. This is what is termed as the Concrete Stage of thinking. It should be an expectation, even in the earliest grades, that students use multiple representations. For example, children might use manipulative materials to model their thinking about a problem, then translate that model to a drawing on paper, and eventually move toward the use of more conventional symbols in expressing their thinking. Multiple representations support students' thinking by allowing them to see the same idea expressed in different ways. With this experience, the students will be able to translate this thinking into a pictorial form by drawing on paper. Adults can then guide the students toward using symbolic representations to express thought by helping them to draw on the connections between the concrete representations and pictorial representations.

It has been suggested that one goal of mathematics instruction is for lessons to occur in a step-by-step manner allowing the learners to move from needing concrete manipulatives to solve a problem to a point where they are able to think abstractly through the steps to solve the problem (Miller and Mercer, 1993). Jean Piaget, the Swiss psychologist, introduced a developmental epistemology that focused on the growth of intelligence from infancy to adulthood. He proposed four stages of cognitive development which reflect the increasing sophistication of children's thoughts:

- **Sensorimotor Stage** (Birth to 2 years of age): During this stage, infants and toddlers acquire knowledge through sensory experiences and manipulating objects. At this point in development, children's intelligence consists of their basic motor and sensory explorations of the world. Piaget believed that developing object permanence or object constancy, the understanding that objects continue to exist even when they cannot be seen, was an important element at this point of development. By learning that objects are separate and distinct entities and that they have an existence of their own outside of individual perception, children are then able to begin to attach names and words to objects.

- **Preoperational Stage** (2 years to 7 years): At this stage, children learn through pretend play, but still struggle with logic and taking the point of view of other people. They also often struggle with understanding the ideal of constancy. During this stage, young children are able to think about things symbolically. This is the ability to make one thing - a word or an object - stand for something other than itself. Thinking is still egocentric, and children have difficulty taking the viewpoint of others.

- **Concrete Operational Stage** (7 years to 11 years): Children, at this point of development, begin to think more logically, but their thinking can also be very rigid. They tend to struggle with abstract and hypothetical concepts. Children can work things out internally in their head (rather than physically try things out in the real world). Children can conserve number (age 6), mass (age 7), and weight (age 9). Conservation is the understanding that something stays the same in quantity even though its appearance changes.

- **Formal Operational Stage** (11 years and above). During this stage, people develop the ability to think about abstract concepts and logically test hypotheses. It should, however, be acknowledged that more recent research has suggested that many teenagers never reached the stage of formal operations (Shayer, Kucheman and Wylam, 1976).

Each child goes through the stages in the same order, and a child's development is determined by biological maturation and interaction with the environment. Although no stage can be missed out, there are individual differences in the rate at which children progress through the stages, and some individuals may never attain the later stages. Piaget did not claim that a particular stage was reached at a certain age - although descriptions of the stages often include an indication of the age at which the average child would reach each stage.

Researchers have explained how features of Piaget's theory can be applied to teaching and learning. Piaget has been extremely influential in developing educational policy and teaching practice. For example, a review of primary education by the UK government in 1966 was based strongly on Piaget’s theory. The result of this review led to the publication of the Plowden Report (1967). The report's recurring themes are individual learning, flexibility in the curriculum, the centrality of play in children’s learning, use of the environment, learning by discovery, and the importance of evaluation of children’s progress - teachers should not assume that only what is measurable is valuable. 'Discovery Learning’ – the idea that children learn best through doing and actively exploring - was seen as central to the transformation of the primary school curriculum.

Because Piaget's theory is based upon biological maturation and stages, the notion of 'readiness' is important. Readiness concerns when certain information or concepts should be taught. As per Piaget's theory, children should not be taught certain concepts until they have reached the appropriate stage of cognitive development. According to Piaget (1958), assimilation and accommodation require an active learner, not a passive one, because problem-solving skills cannot be taught, they must be discovered.

Within the classroom, learning should be student-centered, accomplished through active discovery learning. The role of the teacher is to facilitate learning, rather than direct tuition. Therefore, teachers should encourage the following within the classroom:

- Focus on the process of learning, rather than the end product of it.
- Use of active methods that require rediscovering or reconstructing "truths".
- Use of collaborative, as well as individual activities (so children can learn from each other).
- Devising situations that present useful problems, and create disequilibrium in the child.
- Evaluation of the level of the child's development so suitable tasks can be set.

We know that children begin to develop understanding of mathematical ideas on a concrete level. Physical materials give students an opportunity to express their ideas before they are able to record them with pencil and paper; they also give students the flexibility to make, test and refine their conjectures. This does not imply that once students have developed writing skills, physical models should be eliminated. Many mathematicians and scientists rely on concrete models to test and refine their conjectures!

A successful approach that encapsulates Piaget's links between the concrete, representational and abstract for Mathematics is the CRA strategy. This strategy is an intervention for mathematics instruction that research suggests can enhance the mathematics performance of students in a classroom (Nugroho and Jailani, 2019) as well as of those with Learning Disabilities (LDs), (Bouck, Satsangi and Parks, 2017). It provides a graduated, conceptually supported line of work to create meaningful connections among concrete, representational, and abstract levels of understanding. It is a three-part instructional strategy, with each part building on the previous instruction to promote student learning and retention, and to address conceptual knowledge (American Institute for Research, 2016). The CRA strategy has its roots in the work of Bruner and Kenney (1965), who defined learning through "Stages of Representation":

- **Enactive** – learning through movement and action
- **Iconic** – learning through pictures
- **Symbolic** – learning through abstract symbols

The CRA strategy combines effective components of both behaviourist (direct instruction) and constructivist (discovery-learning) practices (Sealander, Johnson, Lockwood & Medina, 2012; Mercer & Miller, 1992). This strategy is especially effective when used to teach individuals with LDs across grade levels and in many different topic areas in mathematics (Witzel, Riccomini & Schneider, 2008). CRA uses demonstration, modeling, guided practice followed by independent practice, and immediate feedback, which are aspects commonly found in direct instruction. Prior literature reviews have found using direct and explicit instruction for students with LDs in mathematics to have strong effect sizes (e.g., Baker, Gersten & Dae-Sik, 2002; Gersten, Chard, Jayanthi, Baker, Morphy & Flojo, 2009; Zheng, Flynn & Swanson, 2013).

CRA also includes discovery-learning strategies involving representation to help students transition between conceptual knowledge and procedural knowledge (Sealander, Johnson, Lockwood & Medina, 2012). Note that this will be particularly important for children with dyslexia and other learning disabilities, as these have been linked to a deficit in procedural learning, (Nicolson and Fawcett, 2007).

The C-R-A instructional sequence consists of three stages, concrete, representational and abstract:

Concrete: This is known as the 'doing' stage and involves physically manipulating objects to solve a math problem. The teacher begins instruction by modelling each mathematical concept with physical materials also known as concrete manipulatives (e.g. red and yellow chips, cubes, base10 blocks, pattern blocks, fraction bars and geometric figures) not to solve, but rather to visualize the problem or a concept. Students are guided by the teacher to meaningful interactions with the hands-on materials to model the concept or skills.

Representational: It is known as the "seeing" stage and involves using images to represent objects to solve a math problem. The teacher transforms the concrete model into a representational (semi-concrete) level, which may involve drawing pictures; using circles, dots and tallies; or using stamps to imprint pictures for counting. Students draw pictures that represent the concrete objects previously used.

Abstract: It is known as the "symbolic" stage. The teacher models the mathematics concept at a symbolic level, using only numbers, notation and mathematical symbols to represent the number of circles or groups of circles. The teacher and students use operation symbols (+, x, -) to indicate addition, multiplication or subtraction.

As the teacher moves through the concrete-to-representational-to-abstract sequence of instruction, the abstract numbers and/or symbols should be used in conjunction with the concrete materials and the representational drawings. CRA is an interconnected instructional sequence, and these explicit connections between lessons and stages are crucial in order for students to learn the targeted skill as well as comprehend the associated concepts (Witzel, Riccomini & Schneider, 2008). However, the power of C-R-A lies in its ability to help students transform their thinking from the physical to the mental. This instructional sequence is only powerful if teachers are able to help students join the dots between what is similar in the three modes of representation: students' experience with concrete materials, the picture representation, and the symbolic representation.

Mathematics teaching involving the C-R-A teaching sequence often includes the following steps:

1. Teach the math concept using manipulatives (concrete level).
2. Allow ample opportunities for students to practice the concept using various manipulatives.
3. Make sure students understand the concept at the concrete level before moving on to the representational level.
4. Introduce pictures to represent objects (representational level). Model the concept.
5. Provide plenty of time for students to practice the concept using drawn or virtual images.
6. Check student understanding. Do not move to the abstract level if students haven't mastered the representational level.
7. Teach students the math concept using only numbers and symbols (abstract level). Model the concept.
8. Provide plenty of time for students to practice the concept using only numbers and symbols.
9. Check student understanding. If students are struggling, go back to the concrete and representational levels.

Once the concept is mastered at the abstract level, periodically bring back the concept for students to practice and keep their skills fresh.

Modelling the concept and providing lots of opportunities to practice is extremely important at all the three levels. Also, one should not rush through the levels. Students need time to make connections and build on what they already know. It is important to give them the time to process the information before moving on to the next level. The CRA strategy is compliant with Universal Design, as it involves the following:

- Multiple ways to teach Math concepts
- Multiple means of representation offered through the use of various manipulative items, visual images and technology (Smart Board, computer games/software, video.
- Allows options for how students learn and express their understanding of a math concept (assessment example: Use Smart Board clickers to ease student anxiety when having them give answers to math problems. This will, in turn, increase student engagement and participation.)
- Flexible methods for engaging students (able to incorporate student interests and use real life examples)
- Accessible to all students regardless of ability level
- Allows for accommodations to be made
- Learning is active

Many students with LDs struggle to attach meaning to abstract concepts and symbols (Anstrom, n.d.). Introducing a concept using hands-on manipulatives gives the students a concrete way to understand it. This is beneficial not only for the students with LDs, but also for the entire class.

In addition, the CRA strategy is multi-modal, appealing to students whose brain strengths lie in several different areas [multiple intelligences (Gardner, 2011)]. The manipulatives of the concrete stage satisfy the needs and desires of students with a strong bodily/ kinaesthetic intelligence, the drawings and diagrams from the representational stage are able to take account of students with a visual-spatial intelligence, and the numbers and symbols of the abstract stage play to the strengths of students with a logical/ mathematical intelligence. The multiple forms of representation benefit the rest of the student body as well, since regardless of personal strengths and weaknesses, everybody learns better when the concept is reinforced through a variety of media. The manipulatives and visual aids are not just provided for students with disabilities, but instead are an integral part of the lesson for everyone - thus making the concept accessible for students of all abilities and learning styles. The CRA strategy is multisensory (use of all senses simultaneously), accounting for students' senses of touch, sight and hearing. It is the concrete-kinaesthetic-tactile "prove by construction" level that allows the student to create memory other than with words. It is directional, spatial and visual all at once. Thus, it addresses not only the physical manipulation of objects, but also engages the visual and spatial areas of the brain. It creates tactile memory just as movement around one's room creates a spatial memory that can be utilized when the lights go out.

For many students, this portable memory is foundational. They draw on it to "see the math" in applications. They visualize construction and deconstruction of quantity relationships as they apply them across operations.

The representational and abstract stages are visual. As the students progress in age and math education, they are capable of creating more complex visual models. They acquire the fine motor skills necessary to draw solutions to complex problems. They rationalize the way quantities "go together" as they illustrate solutions which also become place holders for language. They use these drawings to justify solutions, and reason through solving problems in which they must defend their results. For some students these drawings become visual cues from which they may recall the steps they took to reach a conclusion. They can use the drawings to rationalize and describe the process. This is at the heart of the new standards-based curricula. Students are asked to describe and defend solutions, and explain why the result they achieved is a reasonable solution. For the students with language impairment, this is a difficult task, and the ability to refer to the drawing may be a better guide than the computations themselves.

All three stages of the CRA strategy have auditory components as well, since the teacher has to explain the concept verbally at each stage. This means that the process demands simultaneous processing, a very useful strategy for teaching mathematics to struggling learners. This means employing as many sensory areas as possible and all at the same time. So, if we consider the child listening to the teacher, and concentrating on their own performance, it can be seen that the pencil point provides a focus to the eye (visual senses), while the internal voice is engaged to talk through a sequence or process (auditory processing). The pencil point anchors the hand in place until the correct word or answer is retrieved before moving on. The internal self-talk that leads a student through multiple steps to a solution also enables that student to retrace those steps because the language is the same, repetitive and reliable. In terms of attention and focus, simultaneous processing is the single most effective way to slow the impulsive student down, to make the inattentive student attend, and the body-voice-internal auditory mechanism sustain performance. By tying as many modalities to a single pursuit, each sensory area restrains the others. The unity of performance is the talk aloud and trace or point. For students with learning differences, it should be practiced aloud before becoming internal. Later, as the processes and procedures become automatic, levels which were previously needed become less important and can be eliminated. It is in the simultaneous performance that automaticity can be improved, rapidly developed, and eventually become computational habits which sustain accuracy.

Great diversity among students can be a challenge for many teachers. English Language learners and students with weak language skills may experience difficulty with new vocabulary, often because the definitions use new words they do not know. For these students, the use of concrete manipulatives and visual imagery is a bridge to meaning. Multisensory methods are truly a gift for these students because the manipulatives and the imagery help to teach new words quickly through association. By using general images to teach mathematical concepts, teachers can help these students grasp the meaning of Maths terms easily. The visual and tactile memories associated with using manipulatives and imagery can guide them through concepts and operations in a way too many new words cannot do.

The predominance of language-based LDs in the population with Learning Disability demands that teachers become knowledgeable about the impact of language-based problems and the associated remedial strategies which help these students succeed. These students require explicit instruction using precise terms that are mathematically accurate. The teacher should strive to reduce the language load and should use a moderate rate of speech with frequent "processing pauses" for the students to comprehend, re-verbalize or associate with a visual or concrete model. These students will benefit from frequent use of summary sheets which tie concepts and procedures together with repetitive, retrievable phrases. They will benefit from "wait time" as an accommodation as they organize their words for a response. These are the students who will benefit from drawing solutions to the problems where appropriate, and using the visual imagery as a road map to expressive language demands.

Benefits of the CRA Strategy (Research-Based Education Strategies and Methods, Making Education Fun, 2012)

- Provides students with a structured way to learn math concepts
- Students are able to build a better connection when moving through the levels of understanding from concrete to abstract
- Makes learning accessible to all learners (including those with math learning disabilities)
- Follows Universal Design for Learning guidelines
- Aligned with National Council for Teaching Mathematics standards
- Multimodal - has kinaesthetic, visual-spatial, and logical/mathematical components.
- Multisensory – able to take account of visual, auditory and kinaesthetic learning styles.
- The approach builds on itself: students first develop a concrete understanding of the concept, and then can use that to bridge to a more abstract understanding (Anstrom, n.d.).
- Manipulatives and visual aids improve students' ability to retain

understanding of the math concepts (Boggan, Harper & Whitmire, 2009)

- The manipulations in the concrete and representational stages allow students to rationalize the conceptual mathematical procedures into logical steps and understandable definitions (Witzel, Riccomini & Schneider, 2008) when students encounter difficulty with representations to assist in finding the solution (Witzel, 2005).
- Research has shown that students who use hands-on methods tend to show more motivation, develop more precise and comprehensive representations, and understand and apply ideas more easily than students who do not (Anstrom, n.d.).
- Manipulatives have been found to decrease math anxiety (Boggan, Harper & Whitmire, 2009).
- Students learn better when they have concrete examples in front of them.
- Using manipulatives in the Maths lesson can help to improve students' interest and curiousity in learning.
- Research performed by the Access Center in 2004, demonstrates that CRA works well in both the elementary and secondary levels, and that it can be used successfully in a classroom setting, a small-group setting, or with individual students (Arefeh, Dragoo, Luke & Steedly, 2008).
- When students are taught using CRA, they have been able to generalize and maintain the progress they made during the CRA intervention period (e.g., Bryant et al., 2008; Sealander, Johnson, Lockwood & Medina, 2012; Witzel, 2005).

In terms of evidence for the effectiveness of this approach, numerous studies have shown the CRA instructional strategy to be effective for students both with LDs and those who are low achieving across grade levels, and within topic areas in mathematics such as:

- basic facts and place value (Bryant et al., 2008; Miller & Mercer, 1993),
- addition, subtraction, multiplication and division (Mancl, Miller & Kennedy, 2008; Miller & Kaffar, 2011; Miller & Mercer, 1993; Sealander, Johnson, Lockwood & Medina, 2012),
- fractions (Butler, Miller, Chrehan, Babbitt & Pierce, 2003; Jordan, Miller & Mercer, 1999; Misquitta, 2011)
- word problems (Hutchinson, 1993; Maccini and Hughes, 2000)
- algebra (Witzel, 2005; Witzel, Mercer & Miller, 2003

With regard to mathematics students (first and third graders), Fuchs, Fuchs, and Hollenback (2007) also advocate the use of the CRA sequence to teach place value, geometry and fractions. Their study consisted of two third-grade classes in which the same geometry unit was presented. The teacher of one class used only charts and drawings to teach the unit concepts, and the teacher of the other used manipulatives. The class that used manipulatives scored "significantly higher" on the test both groups took at the end of the unit (Boggan, Harper & Whitmire, 2009). This suggests that, perhaps, the concrete stage of the CRA strategy is vital in terms of having physical experiences to build conceptual understanding. (Butler et al., 2003). However, it is important to keep in mind that the concrete stage is not the only stage. Studies have also proven that when teachers introduce manipulatives as an exercise completely separate from the abstract math concept, students do not make the connection between the manipulatives and the abstract math (Boggan, Harper & Whitmire, 2009). More recently, Flores and colleagues (Flores et al, 2018) found that CRA intervention was successful for 5th grade children receiving tier two interventions for fractions, and Peltier and colleagues established that students with emotional and behavioural difficulties who struggle with abstract representations of number, can be successful using the concrete manipulatives of CRA (Peltier and Vannest 2018).

One benefit of the CRA approach is that teachers can develop their own CRA methodology adapted to the topic they wish to enhance. Witzel, Riccomini and Schneider (2008) developed an acronym that teachers can use to assist them in creating their own CRA instructional sequence.

CRAMATH outlines **seven steps** teachers can use to create a mathematical unit:

1. **C**hoose the math topic to be taught.
2. **R**eview procedures to solve the problem.
3. **A**djust the steps to eliminate notation or calculation tricks.
4. **M**atch the abstract steps with an appropriate concrete manipulative.
5. **A**rrange concrete and representational lessons.
6. **T**each each concrete, representational and abstract lesson to student mastery.
7. **H**elp students generalize what they learn through word problems. (p. 273.).

The 'Concrete' segment of CRA, in particular, has been the theoretical basis for the use of manipulatives in learning mathematics (Reisman, 1982; Ross & Kurtz, 1993). According to the research cited by Terry Anstrom (n.d.), "students who use concrete materials develop more precise and more comprehensive mental representations, often show more motivation and on-task behavior, understand mathematical ideas, and better apply these ideas to life situations." Research shows that using the CRA strategy is very effective for students who have a learning disability in math (Anstrom, n.d.). A study performed in 1993 showed that CRA instruction helped students with LDs acquire and learn basic mathematics skills. Research suggests that this was because CRA ensures that students have a firm understanding of the underlying concepts of math before they learn the "rules" (Anstrom, n.d.).

In studying the effects of using manipulatives and the CRA instructional sequence, Ojose and Sexton found that: "The importance of providing students with direct experiences with concrete material is supported by evidence from the classroom and an understanding of how learning takes place. While children can remember information taught through books and lectures, studies show that deep understanding and the ability to transfer and apply knowledge to new situations requires learning that is founded on direct, concrete experience. Research shows that using manipulatives in conjunction with other methods can deepen students' understanding of abstract concepts" (Ojose and Sexton, 2009). They advocate the use of manipulatives to strengthen a student's understanding of concepts and provide a direct link to meaning. This is certainly necessary for those students for whom abstraction itself is a challenge.

Representations are valuable tools in problem solving, reasoning, and communicating about mathematical ideas. Young students can and should use a variety of representations to show and explain their thinking. We should also remember that representations are not an end in themselves, but rather a fundamental part of the process of learning mathematics.

Young children enter school with limited ability to express ideas in writing. With formal language instruction in the early grades, students begin to develop the ability to express themselves mathematically by using written words and symbols. However, young children enjoy drawing pictures regardless of their writing skills. They move naturally from the physical representations to written representations by drawing pictures of mathematical ideas. Young children often invent ways to represent mathematical actions.

As students progress through the early grades, they should continue to express their thinking with pictures and (by second grade) with verbal explanations to describe their pictorial representations. The transition from pictures to more abstract mathematical symbols should be an integral part of students' representations, but it should not be forced on the students. For some students, the connection to numeric and operational symbols comes easily. Others take a longer time to use abstract symbols to represent their ideas. It is important to remember that the goal here is understanding of those symbols and not a rote memorisation of abstract symbols. Inventing strategies, using models and pictorial and symbolic representations, and explaining their ideas should be an essential part of young children's mathematics experience.

The job of a teacher is not to "cover" a list of mathematical concepts, but rather to give students ample time and opportunities to discover the relationships between their world and the world of mathematics, and the meaning of these relationships.

The gifted student may also have LDs, display impulsivity, or be inattentive. Structured procedures and repetitive exercises can lead to automaticity and absence of words, or this student will sub-vocalize procedures coded with specific words as cues which occur automatically and lead to rapid, but accurate computations. An example would be in solving equations, another would be the long division algorithm. By sub-vocalizing repetitive questions, the student with LDs can be fully competent with multi-step procedures. Structured procedures can organize mental chaos, and encourage real math reasoning as the student is freed from the tedium of the computation words.

The gifted student without LDs must also be challenged. Engaging applications and problem-solving activities can encourage creativity. Many of these students enjoy challenges such as exploring engineering concepts through geometry, building bridges with craft sticks, and discovering which geometric shapes are the sturdiest for construction. They might demonstrate applications in robotics or building construction.

To deal with the diversity of learners in each classroom, the teacher must begin to think outside the book. Games are a great way to practice skills maintenance and fluency, and students with disabilities can be given games as a way to practice restricted numbers of facts.

All students will benefit from a multisensory inclusion approach. It is appropriate for all though essential for some. All students can be offered academically appropriate materials in grade level content using simple modifications in format and facts. All students will benefit from inclusion and collaboration in problem solving activities which encourage both expressive and receptive language in small groups.
Using restricted number facts for new introductions and then providing computationally appropriate practice through differentiation allows all students to apply grade level concepts in meaningful ways.

To conclude, the CRA strategy for teaching mathematics, stemming from the socio-constructivist paradigm, is an excellent instructional approach when used correctly and followed exactly. For the strategy to be effective, the teacher must reinforce the connections between the concrete and representational stages, and then between the representational and abstract stages. There is little to be gained if each stage is completely isolated from each other, and the linkage is never explained. However, if the students understand that the manipulatives, the pictures, and the written maths equation are always representing the same problem, they will come out with a firm understanding of the meaning behind the maths, and will have multiple strategies from which to choose when it comes to solving that kind of problem. It is particularly important here to emphasize the role of the teacher in helping to facilitate learning by providing guidance and direct instruction for the students' exploration, and ensuring that students pay attention to key features that are common in the Concrete-Representational-Abstract mode.

SUMMARY

As per Piaget's Four Stages of Mental Development, till 11 years of age, children are at the concrete operational stage - thinking still tends to be tied to concrete reality, and ideas are obtained from action on concrete objects. The National Council of Teachers of

Mathematics, USA, makes it known that all students benefit from the use of manipulatives and visual aids.

The Concrete-Representational-Abstract strategy is an intervention for mathematics instruction that research suggests can enhance the mathematics performance of students in a classroom as well as of those with learning disabilities. The CRA strategy uses demonstration, modeling, guided practice followed by independent practice, and immediate feedback, and is compliant with the Universal Design for Learning. Learning at the concrete stage becomes more meaningful when it is multisensory in nature, i.e., all learning pathways in the brain are used simultaneously. The strategy is effective when students make the connection between the concrete and representational stages, and then between the representational and abstract stages.

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