Asia Pacific Journal of Developmental Differences
Vol. 10, No. 1, January 2023 pp. 5– 26
DOI: 10.3850/S2345734123000182

Prevalence and characteristics of geometric difficulties in elementary school children

Mirela Duranovic[1*] and Edna Didic[1]

1. *University of Tuzla, Bosnia & Herzegovina*

Abstract

The current study aimed to analyze the prevalence and characteristics of geometric difficulties in elementary school children. In cooperation with teachers, tasks for assessing geometric knowledge, respecting the curriculum for a particular grade, have been developed. The level of geometric thinking was analyzed as an additional factor for classifying geometric difficulties and for better understanding problems that can lead to determining appropriate accommodations. The prevalence of geometric difficulties was 9.2% and students with geometric difficulties were on the first and second level of geometric thinking. Deficits in visual-spatial skills have been also analyzed as potential risk factor for developing geometric difficulties.

Keywords: geometry, geometric level, geometric difficulties, visual-spatial perception, visual working memory

** Correspondence to:*
Dr Mirela Duranovic, Department of Speech and Language Pathology of the Faculty of Education and Rehabilitation, University of Tuzla, Bosnia & Herzegovina. Email: mirela.duranovic@gmail.com

INTRODUCTION

Mathematical thinking is involved in almost all aspects of modern life. Consequently, people with poor mathematical skills have problems graduating high school, going to college, having a steady job (Bynner & Parsons, 2006). It is estimated that 3–6% of the population suffer from the specific mathematics learning disability called developmental dyscalculia (DD) (Badian, 1999; Gross-Tsur et al., 1996). The latest DSM-V diagnostic criteria define DD as a neurodevelopmental disorder characterized by difficulty in learning about numbers and arithmetic, which manifests in children despite adequate neurological development, intellectual abilities and schooling opportunity (American Psychiatric Association, 2013). Individuals with DD exhibit an inability to place basic arithmetic information in long-term memory, to understand or access quantities connected with number words and Arabic numerals, and have problems in learning arithmetic procedures (Butterworth, Varma & Laurillard, 2011; Mazzocco, Feigenson & Halberda, 2011).

There are four basic areas of mathematical skills: number processing; arithmetical procedures; arithmetic facts retrieval; and geometrical abilities (Dehaene, 1997; Geary & Hoard, 2001; McCloskey, 1992; Rubinsten & Henik, 2009; Von Aster & Shalev, 2007), with different cognitive and neural correlates, different influences of neurological processes and environmental factors, leading to different subtypes of DD (Pedemonte et al., 2022).

Karagiannaki, Baccaglini-Frank and Papadatos (2014) proposed a classification model for mathematical learning difficulty, including four subtypes: core number, memory (retrieval and processing), reasoning and visual-spatial. The Visual-spatial subtype includes the domains of written arithmetic, geometry, algebra, analytical geometry and calculus (Geary, 1993, 2004; Mammarella et al., 2010; Rourke & Conway, 1997; Venneri et al., 2003).

Although geometry is the part of mathematics in which the properties of space and objects in space were originally studied (Volenac, 1979), fundamental components of mathematics learning (Final Report of the National Mathematics Advisory Panel, 2008), less attention was paid to it in research on mathematical difficulties. In a systematic review of mathematics interventions for students with learning disabilities (Maccini, Mulcahy, & Wilson, 2007), only one study (see Cass et al., 2003) included achievements in geometry. No research about interventions in geometry was found in a meta-analysis of studies of mathematics interventions for elementary students with special needs (Kroesbergen & Van Luit, 2003). Chew and Lim (2013) pointed out the importance of learning geometry as a basic skill for learning other topics in mathematics such as fractions, decimals, percentage, functions and calculus. For appropriate knowledge in mathematics and science, all children should learn geometric shapes and spatial relationships, use visualization and spatial reasoning to transform shapes, and apply geometric modeling to solve problems (NCTM, 2000), therefore geometry should not be ignored.

Wong, Hsu, Wu, Lee, and Hsu (2007) noted that geometry is more difficult than other mathematics areas and geometric problem solving can be especially challenging for students because of problems with the comprehension of geometric problems. Therefore, a very important question that needs to be open is do we have children with geometric difficulties in schools, what criteria to use to classify geometric difficulties, what is the prevalence of geometric difficulties and which risk factors lie behind these difficulties. The current study tried to find answers to all of these questions.

Prevalence of Mathematic Disorder

Knowledge of the prevalence of learning disabilities enables the determination of the extent of learning disabilities in the normal population of school children, recognition of risk factors, and helps develop therapeutic strategies. The law requires the provision of services for individuals with learning disabilities, and public health issues are also important. Data on prevalence, relative risk, outcome, and effective therapeutic procedures are necessary to make appropriate decisions to secure the resources needed for medical and educational services (Hammill, 1990).

To determine prevalence we must develop a scientific and clinical consensus as to what constitutes a learning disability and which definition best describes the problem. Prevalence studies on DD have been carried out in various countries using different definitions. In spite of the lack of definitional consistency, the prevalence of DD across countries is relatively uniform, ranging from 3-6 % in the normal population (Shalev, Auerbach, Manor & Gross-Tsur, 2000). Kosc (1974) in Bratislava and Badian (1983) in American children found prevelance at 6.4 %. In England 1.3 % children had a specific arithmetic disability and 2.3 % arithmetic and reading disabilities (Lewis, Hitch & Walker, 1994).

Geometry is, with arithmetic, one of the oldest branches of mathematics. It is concerned with properties of space that are related with distance, shape, size, and relative position of figures (De Risi, 2015). Geometry encompasses many wider areas of problems, practical and abstract, which may have practical application (Volenac, 1979). Learning geometry helps students to develop skills such as visualization, critical thinking, intuition, perspective, guessing, deductive reasoning, logical reasoning, and proving (Šutalo, 2016). Learning disability can include mathematics difficulties which result from problems in cognitive skills that have influence on student's ability to represent or process information in one or all of the mathematical domains, such as geometry (Geary, 2004). It has been noted that deficits in geometric abilities can be manifested by visuo-perceptual mistakes in symbol processing, arranging numbers for written calculations, understanding graphs and figures, and identification of visual characteristics of objects (Geary, 1993). Symptoms can be manifested in the processing of visual-spatial information, including spatial orientation, directions and distances, and the transformation of three-dimensional objects (Kinach, 2012; Simic et al., 2013).

Knowing the prevalence of geometric difficulties would be of great importance and would have a variety of clinical, educational, and public health implications. Determining prevalence would help in analyzing the success of educational programs and teaching methods. It would be important for agencies responsible for providing medical services and special educational interventions (Shalev, 2007).
t

The Theory of the Level of Geometric Thinking

The model that the best defines student levels of geometric thinking level is based on Van Hiele's model (Battista, 2002). The theory of the Dutchman Pierre van Hiel explains why a large number of students have problems with geometry, especially with formal proofs. His theory consists of five levels of thinking through which students will gain the ability to perform formal evidence and understanding (Ovčar, 1990). At each subsequent level, new knowledge is acquired, and in order to reach the next level, it is necessary to adopt the previous one. It depends solely on the understanding of a particular material and the perception of the whole concept, and not only on the acquisition of factual knowledge. Also, age does not affect the transition to the next level. There are people who have remained at the initial level throughout their lives despite going through the school system within which geometric content was processed (van Hiele, 1986).

According to Van Hiele (1986), five levels have been explained:

- **Level 1 - Visualization Level.** Students, at that level, make decisions based solely on the basis of perception, without knowing any reason. They are able to recognize geometric shapes, such as: triangle, quadrilateral or circle, but their properties are not known to them, and they often believe that something is just like that based on one example only.

- **Level 2 - Level of Analysis.** Students, at this level, see figures as a set of properties, and learn terms to describe them, but they still cannot see the connection between them. When describing an object, they list all its properties, but they cannot distinguish - which of them are necessary and which are sufficient to describe it. They can draw conclusions inductively, based on a few examples, but they still cannot use deduction. They begin to believe that if a figure belongs to a class of squares, then it has all the properties of that class, such as, for example: mutually normal diagonals, sides of equal length, right angles, lines of symmetry, and other properties.

- **Level 3 - The Level of Abstraction, or the Level of Informal Deduction.** Students learn about the relationships between the properties of geometric shapes and, based on that, the relationships between the geometric shapes themselves. They begin to think deductively, but do not yet understand the rule and meaning of formal deduction. At this level, students begin to think about what is needed and what is enough to describe a geometric figure. For example, they know that it is enough that a quadrilateral, which has all sides equal, has one right angle, to be a square.

- **Level 4 - Deduction Level.** Students can derive high school-level evidence, draw conclusions from previously known claims, understand the meaning of definitions and axioms, and understand the meaning of a necessary and sufficient condition. Students are able to use abstract concepts, and to draw conclusions that are based more on logic than on intuition.

- **Level 5 - Level of Strictness.** At this level, older students are able to understand the consistency, independence and completeness of the axiomatic system, and to compare mathematical systems. They can also understand indirect proof - proof using contraposition, and understand non-Euclidean geometric systems.

In the original works van Hiele's numbers were numbered from 0 to 4, while in some American theories based on van Hiele's theory the levels are numbered from 1 to 5, while level 0 is complete ignorance of any form of geometry (Crnjac, 2013). The order of the levels of thinking that students go through is invariant, on the way to the highest level, no previous level can be skipped which was shown in the studies conducted by Burger and Shaughnessy (1986), and Fuys, Geddes and Tischler (1988).

It is difficult to determine the time it takes to move from one level to the next, because only active learning and work on understanding the material is important, and some need more and some less time (Crnjac, 2013). According to van Hiele’s theory, the main reason why a traditional curriculum fails is that it is presented at a higher level than that at which students function. In other words, students do not understand the professor, nor does the professor understand why students do not understand him (De Villiers, 2008).

It is desirable to examine the level of students, so that they are not taught material that they are not able to adopt, even if they make an effort and listen. This can be done with the tests which are intended for assessing knowledge needed for each level (Bilbija, Milanković & Runjić, 2009). This study had intention to analyze at what level are students in elementary school, especially students with geometric difficulties.

Visual and Spatial Domain and Achievement in Geometry

Mathematics is very complex and includes different areas that integrate different abilities related to the sense of quantity, symbols decoding, memory, visuospatial capacity, logics, to name a few. Problem in any of these abilities may lead to mathematical learning difficulties (Karagiannakis, Baccaglini-Frank & Papadatos, 2014).

The connection between visuospatial skills and mathematics learning disabilities has not been sufficiently explored (Geary, 2004). Studies were mostly caried out with adult or neuropsychological populations, and using that result with the developing skills of children must be made with caution (Bull, Johnston, & Roy, 1999). Visuospatial areas support different mathematical competencies, including the specific domain of geometry and the solving of complex word problems (Dehaene et al., 1999; Geary, 1996). Therefore, problems in visuospatial skills could affect the occurrence of a corresponding learning disability (Geary, 2004).

Karagiannakis et al., (2014) analyzed the literature as well as unpublished clinical observations and proposed a classification model for mathematical difficulties. Four basic cognitive domains have been described including visual-spatial as subtypes. Geometry was connected only with deficits in visuo-spatial working memory and the reasoning/ perception specific systems. For this reason, it is very important to study the relationship between visual spatial working memory (and a particular visual-spatial domain) and achievement in geometry.

Geometry in teaching is the best tool for developing mathematical thinking (Jozić, 2008). It provides students with an aspect of mathematical thinking that is different from the world of numbers, but also related to it. As students become familiar with characters, bodies, locations, transformations, and develop spatial thinking, a foundation is laid for understanding not only the spatial world but also other topics in mathematics and the arts, science, and social research (Razel & Eylon, 1991).

Working memory is a system of limited capacity that allows temporary storage and manipulation of information (Baddeley, 2000). Giofrè, Mammarella, Ronconi, and Cornoldi, (2013) have shown that working memory can be associated with geometric achievements in late adolescence. Numerous studies have shown that working memory predicts success in school, on tasks such as reading comprehension (Daneman & Carpenter, 1980), mathematical success and arithmetic problem solving (Passolunghi & Pazzaglia, 2004). Previous research has shown that geometry includes visual-spatial working memory, the ability to retain and manipulate visuospatial information (Giofrè, Mammarella, Ronconi, & Cornoldi, 2013).

Visuospatial working memory can predict a person's success in geometry-related activities. Geometric problems usually require determining a solution for the problem and this ability refers to higher order control (Clements & Battista, 1992). In fact, visuospatial abilities, are very important in geometric achievements. Also, some researchers have shown that children with mathematical difficuties experience deficits in visuo-spatial sketchpad (e.g., Cai et al., 2013; D'Amico & Guarnera, 2005; McLean & Hitch, 1999).

Purpose of the Study

In order to better understand geometric difficulties in elementary school children, the aim of this study was to determine the prevalence of geometric difficulties in students from third to fifth grade of elementary school, their geometric level, and the relationship between visual-spatial perception, working memory and achievement in geometry. We hypothesized that the prevalence of geometric difficulties will be comparable to the already known prevalence of mathematical difficulties, geometric level would be appropriate additional factor in the determination of children who have geometric difficulties, and visual and spatial deficits are risk factors for developing geometric difficulties.

METHOD

Sample

The study included 120 students from primary schools, 65 boys and 55 girls, aged 9 to 11 years (from third to fifth grade). Random sampling was used as a technique in which each sample has an equal probability of being selected. A randomly selected sample should be an unbiased representation of the total population.

First, 120 students were tested with the aim of determination those who have difficulties in geometry. Eleven children were determined to have geometric difficulties. After that the control group of 11 participants was randomly selected, and consisted of children the same age and gender as the group of students with geometric difficulties. The research was conducted in primary schools in the Brčko District in Bosnia and Herzegovina. The subjects were examined individually in quiet room.

A Raven's progressive color matrix test (PMBs) (Raven, 1956) was used by a qualified psychologist with the aim of excluding children with intellectual disabilities. PMBs are one of the standard tests of nonverbal intelligence. PMBs consists of three series with 12 items: A, AB and B. Within each series, the items are approximately ranked by difficulty, so that they are organized from easier to harder tasks. The task of the respondents is to choose the item that completes the "empty" part of matrix from the offered answers. The time of testing was not limited. No children with intellectual disabilities were found.

Instrument

The Test for assessing geometry skills was prepared according to the curriculum for a given class in cooperation with the teacher. A team of elementary school teachers prepared, reviewed the test and and agreed regarding the most suitable items. Each of the tests, for a particular grade (the third, the fourth and the fifth grade), contained of 10 tasks. Participants had to choose the correct answer, complete the answer or connect the correct answers. The test included the different tasks for different grades, for the third grade: lines, marking a line, circle, diameter, and angles; for the fourth grade: line, length, mathematical signs, angle, triangle, and circumference; for the fifth grade: area of squares, rectangles, squares and cubes, conversion of units of measurement, and geometric bodies (See tests in Appendix A). Two standard deviations below the age-specific mean was used as deficit criteria.

On the test for assessing geometry skills, the students who scored below two standard deviations of the mean performance were classified as having difficulties in geometry. In total, 11 students were determined as those with geometric difficulties. (See Table 1 for participant characteristics and group differences).

Table 1. Test Scores for Children With and Without Geometric Difficulties

Variable	students without geometric difficulties		students with geometric difficulties		F	p
	M	SD	M	SD		
Age	10.76	.72	10.74	.78	.00	NS
Geometric skills[a]	6.73	1.27	2.00	1.00	93.89	<.001
IQ[b]	32.18	1.94	31.36	1.91	.99	NS

Note. Test for assessing geometric skills; bIQ, average of individual intelligence quotients according to the Standard Progressive Matrices Test.

The next task included a geometry test that covers all 5 stages of the development of geometric thinking based on Van Hiele model (1986). The test included the following tasks: basic geometric shapes that students should recognize; basic geometric shapes by which students should determine properties or attributes, type, classify and draw them; tasks in which students are able to establish relationships between the properties of shapes, recognize similar shapes, make assumptions and intuitively make simple conclusions; a task that determines whether children understand the meaning of definitions and axioms, make conclusions from previously known statements; a task to determine whether children are able to form connected sets of conclusions and thus justify their thinking, to show whether they can make conclusions. Each level has five questions. If the students answers three or more questions correctly, they have reached the particular level. The same criteria was used by Usiskin(1982) with the passing rate at 60%.

After seven days from the first part of testing, the second part of the research was conducted, which included nine tasks presented in the Psychology Experiment Building Language (PEBL software, Mueller & Piper, 2014; Mueller & Esposito, 2014), related to visual-spatial perception and visual memory. The tests are freely modifiable and available via the GNU Public License, Version 3 (GPLv3). The average examination time for one student was 45 minutes.

The Bivalent Shape Task (BST) was used for testing the ability of visual-spatial perception. One shape, a blue circle or a red square, appears in the middle of the screen. The test required from the participant to determine whether a shape at the center of the screen is a circle or square. Circles are always responded to with the left response, and squares are always responded to with the right response. Visual response cues are provided below the stimulus, indicating the side of the response. However, these response cues are shaded in either red or blue. In all cases, color is irrelevant and not used to make the decision. The stimulus shape is presented either in red, blue, or an unfilled black outline. Thus, three basic trial types exist: congruent trials, in which the irrelevant color of the stimulus matches the response cue; neutral, in which the stimulus is black and white, and incongruent, in which the (irrelevant) color mismatches the response cue. Dependent measures of interest are the speed and accuracy with which participants are able to make the decision (Mueller & Esposito, 2014).

The Clock Test is a test of sustained visual attention. It is an implementation of the so-called "Mackworth Clock Task". Participants have to watch a clock and determine when it skips a beat. The Mackworth Clock is an experimental device used in the field of exerimental psychology to study the effects of long term vigilance on the detection of signals (Mackworth, 1948).

The Corsi block-tapping test is a psychological test that assesses visual-spatial short term working memory. It involves mimicking a researcher as they tap a sequence of up to nine identical spatially separated blocks. The sequence starts out simple, usually using two blocks, but becomes more complex until the subject's performance suffers. This number is known as the Corsi Span, and average is about 5-6 for normal human subjects (Kessels et al., 2000).

The Line judgment task assesses the ability of visual-spatial perception. The student had to determine which of the two lines is longer, pressing the button on the left or the right side. There was a time limit, the red timer measured the time.

Luck and Vogel's multi-object visual working memory task addressed the measurement in the change detection paradigm, popularized by Luck and Vogel (1997). Participants had to detect a change in a visual display containing a varying number of items. Although there were dozens of versions of the task using different types of stimuli, the most popular version (colored shapes) has been implemented in PEBL. The student had to remember the screen full of shapes in different colors. Color shapes appeared on the screen, then disappear and reappear. The new screen could be the same as the previous one or different. The student had to determine whether a difference has occurred or the shapes have remained the same.

The Object judgment test was described in Mueller (2010). The test briefly displays a randomly-generated Attneave shape (see Attneave & Arnoult, 1956; Collin & McMullen, 2002), and after a brief interval, the participant must choose which of two alternative shapes was presented: the original or a slightly altered foil. In this task the ability of visual memory is examined. A figure is displayed on the screen, the student should remember it, because the display disappears, and then a couple of figures is displayed. One of the new figures is identical to the previous one; others are different in size, position. The student should choose the one that is identical to the previous one.

The Mental rotation test examines the ability of visual-spatial perception. It is a simple implementation of Shepard and Metzler's classic mental rotation task (1971). Images of two three-dimensional objects rotated in space are presented. Participants decided whether two images matched or not regardless of their orientation.

RESULTS

One of the goals of the study was to determine the percentage of students with geometric difficulties among elementary school children from the third to the fifth grade. Testing included 120 children. Results showed that 11 students scored below two standard deviations of the mean performance of the controls on the test for assessing geometry skills and they were determined as children with difficulties in geometry. It was noted that 9.2% of elementary school children from the third to the fifth grade had a geometric difficulty. As children get older, difficulties became more pronounced. Among children with geometric difficulties 46% of them were in the fifth grade, 36% in the fourth grade and 18% in the third grade. Most children could solve only one or two tasks, although these tasks have been prepared together with teachers in accordance with the curriculum for a particular class.

The Mean for reaction time on the tasks of the ability of visual-spatial perception and visual memory of children with geometric difficulties and scores produced by the control group are shown in Table 2. A One-way ANOVA was undertaken separately for each task, involving the two groups. It can be seen that the performance of the children with geometric difficulties was slower than that of the controls, for the Bivalent Shape Task that tests the ability of visual - spatial perception, the Corsi block-tapping test that assesses visual-spatial short term working memory and Luck and Vogel's multi-object visual working memory task, but reaction time was faster on the Clock Test of sustained visual attention. No significant effects in reaction time have been found on other tasks. Table 3 shows the accuracy on the tasks of the ability of visual-spatial perception and

Table 2. Mean performance for reaction time in milliseconds on the tasks of the ability of visual-spatial perception and visual memory

	students without geometric difficulties		students with geometric difficulties		F	p
	M	SD	M	SD		
BST	1265.47	638.56	1736.12	830.49	F(1.44)=44.40	<.001
Clocktest	388.78	150.08	354.99	148.84	F(1.44)= 75.64	<.05
Linejudgment	1443.79	836.79	1498.05	1137.96	F(1.44)= .33	NS
Rotation	1320.42	810.02	1473.30	838.10	F(1.44)= 3.79	NS
Corsi	4196.89	1566.74	4841.12	1961.93	F(1.24)=8.12	<.01
Luckvogel	1217.68	611.58	1421.62	992.91	F(1.33)= 5.05	<.05
Objectjudgment	1438.28	890.52	1375.41	1094.19	F(1.44)= .44	NS

visual memory of children with geometric difficulties and the control group, and differences based on the chi-square test. There was significant difference in all tasks between the two groups except on the Corsi block-tapping test. In the group of children with geometric difficulties less correct answers were made than in the control group.

In Table 4 a linear regression analysis was performed to find out which of the visual-spatial perception and visual memory skills (covariates) predicted for the group variable (geometric difficulties vs. skilled geometry skills). Results showed that the geometry skills were primarily explained by the correct answer in the task mental rotation which measures the ability of visual-spatial perception (change of variance = 28.0 %, $p < 0.001$), the correct answer for the object judgment test which measures visual memory (change

Table 3. Accuracy on the tasks of the ability of visual-spatial perception and visual memory

Variables	students without geometric difficulties		students with geometric difficulties		X^2	p
	N (%)		N (%)			
	incorrect	correct	incorrect	correct		
BST	35 (15.9)	185 (84.1)	108 (49.1)	112 (50.9)	55.21	<.001
Clocktest	57 (25.9)	163 (74.1)	141 (64.1)	79 (35.9)	64.79	<.001
Linejudgment	33 (15.0)	187 (85.0)	122 (55.5)	98 (44.5)	78.90	<.001
Rotation	53 (24.1)	167 (75.9)	170 (77.3)	50 (22.7)	124.47	<.001
Corsi	37 (27.8)	96 (72.2)	35 (31.0)	78 (69.0)	.29	NS
Luckvogel	22 (13.3)	143 (86.7)	95 (57.6)	70 (42.4)	70.57	<.001
Objectjudgment	39 (17.7)	181 (82.3)	153 (69.5)	67 (30.5)	120.09	<.001

of variance = 27.0 %, $p < 0.001$), the correct answer for the Luck and Vogel's multi-object visual working memory task (change of variance = 21.0 %, $p < 0.001$), the correct answer for the line judgment task which assesses the ability of visual-spatial perception (change of variance = 18.0 %, $p < 0.001$), the correct answer for the Clock Test which was a test of sustained visual attention (change of variance = 15.0 %, $p < 0.001$), the correct answer for the BST which tests the ability of visual-spatial perception (change of variance = 13.0 %, $p < 0.001$), the reaction time for the BST which tests the ability of visual-spatial perception (change of variance = 9.0 %, $p < 0.001$), the reaction time for the Corsi block-tapping test that assesses visual-spatial short term working memory (change of variance = 3.0 %, $p < 0.05$), the reaction time for the Luck and Vogel's multi-object visual working memory task (change of variance = 2.0 %, $p < 0.05$), and the reaction time for the Clock Test (change of variance = 1.0 %, $p < 0.05$).

Table 4. Linear regression for the dependent group variable and the covariates (visual-spatial perception and visual memory measures)

Predictors	Change of R^2	β	B	SE_B	t	p
BST (rt)	.09	.00	.30	.00	6.66	.00
BST (correct answer)	.13	-.35	-.38	.05	-7.93	.00
Clocktest (rt)	.01	-.11	.00	.00	-2.37	.02
Clocktest (correct answer)	.15	-.38	-.39	.04	-8.70	.00
Linejudgment (rt)	.00	.03	.00	.00	.57	.57
Linejudgment (correct answer)	.18	-.42	-.44	.05	-9.78	.00
Rotation (rt)	.01	.09	.00	.00	1.95	.052
Rotation (correct answer)	.28	-.53	-.53	.04	-13.14	.00
Corsi (rt)	.03	.18	.00	.00	2.86	.01
Corsi (correct answer)	.00	-.04	-.04	.07	-.54	.59
Luckvogel (rt)	.02	.12	.00	.00	2.25	.03
Luckvogel (correct answer)	.21	-.46	-.48	.05	-9.45	.00
Objectjudgment (rt)	.00	-.03	.00	.00	-.66	.51
Objectjudgment (correct answer)	.27	-.52	-.53	.04	-12.82	.00

Table 5 shows the level of geometric thinking of students with geometric difficulties, as well as students without geometric difficulties. At the first level of geometric thinking were 45.5% of students, while at the second level of geometric thinking were 54.5% of students with geometric difficulties. Thus, students with geometric difficulties were at the first and second level of geometric thinking. Furthermore, 81.8% of students without geometric difficulties were at the third level of geometric thinking. At the fourth level of geometric thinking there was only one student, or 9.1% of students without geometric difficulties, which was the case for the fifth level too. Results of the $\chi 2$ test showed a significant effect ($\chi 2$ (4) = 22,000, $p < 0.01$) between students with geometric difficulties and the control group in the level of geometric thinking.

Table 5. Geometric level of thinking in students with and without geometric difficulties

	Geometric level of thinking					Total
	1st level	2nd level	3rd level	4th level	5th level	
Students without geometric difficulties	0	0	9	1	1	11
	0.0 %	0.0 %	81.8 %	9.1 %	9.1 %	100.0 %
Students with geometric difficulties	5	6	0	0	0	11
	45.5 %	54.5 %	0.0 %	0.0 %	0.0 %	100.0 %
Total	5	6	9	1	1	22
	22.7 %	27.3 %	40.9 %	4.5 %	4.5 %	100.0 %

DISCUSSION

The main goal of this study was to determine the prevalence of children with difficulties in one mathematical domain: geometry, to analyze the level of geometric thinking as an additional factor for classifying geometric difficulties and for better understanding problems and to investigate deficits in visual-spatial skills as a potential risk factor for developing geometric difficulties.

Results showed that 7.2% of elementary school students, 9 to 11 years old, have geometric difficulties. In the literature we can find information about the incidence for dyscalculia in general but there is no information about the percentage of students who have geometric difficulties only. The DSM-5 (American Psychiatric Association, 2013) estimates the prevalence rate of 3–7% for deficits in mathematics. Morsanyi, Bers, McCormack and McGourty (2018) used the DSM-5 diagnostic criteria to identify children with a potential diagnosis of specific learning disorder in mathematics (SLDM or developmental dyscalculia) and found prevalence of 5.7%. But DSM-5 diagnostic criteria includes substantial and quantifiable difficulties in learning and using mathematics skills without specifying geometry skills, just stating the arithmetic skills. Wadlington and

Wadlington (2008) noted incidence up to 8% for learning difficulty in mathematics, similar to this research, but also without including geometric skills.

There is increasing evidence showing that students with mathematical difficulties have problems with learning geometry due to visual-spatial deficits (Mistretta, 2000). Results of this study showed that students with geometric difficulties had problem with solving BST, a task which is very easy, and includes the ability of visual-spatial perception of the shape independent of the color. They also had problems to decide which of two lines is longer on the line judgment task. These were unexpected results if we consider that almost half of the students with geometric difficulties were on the first visualization level, and obviously had problems to recognize correct geometric shape and problems to determine the length. They also had problems with solving the Clock test which indicate problems with sustained visual attention, which could be connected with proper solving all other tasks. Many geometry skills depend on spatially representing mathematical relations, and students who struggle in mathematics frequently misunderstand spatial information (Geary, 2003).

Visual working memory supports many mathematical competencies, including geometry (Geary, 1996; Jeung, Chandler, & Sweller, 1997). Visual working memory has not been researched as frequently as verbal working memory in children with mathematics difficulties (Zhang, Ding, Stegall & Mo, 2012). In this study three tasks that measure visual memory were used. Children with geometric difficulties had problems in speed or accuracy during solving these tasks. They were also good predictors of geometric skills. This is in line with previous studies about the connection between geometry and visual-spatial working memory which showed that visual-spatial working memory directly predicts academic success in geometry (Giofrè, Mammarella, & Cornoldi, 2013; Giofrè, Mammarella, Ronconi & Cornoldi, 2013).

According to Hoffer (1981) there are five fundamental skills for mastering geometry: visual, verbal, drawing skills, logical skills, and applied skills. There is evidence showing that students with geometric difficulties have difficulties representing visual shapes (Triadafillidis, 1995), which was noted in this study too. Most students with geometric difficulties were on the second level, they saw figures as a set of properties, and learnt terms to describe them, but they cannot make connection between them.

Existing research rarely has addressed three-dimensional geometric properties and visual spatial imagery, although students are frequently less proficient with three-dimensional geometric shapes than they are with two-dimensional shapes (NRC, 2001), therefore we used tasks with three-dimensional geometric shapes. Students with geometric difficulties differ from their peers in the ability to mentally rotate one figure to compare it with another, to decide whether or not two objects they see are identical. Correct answers on this task was a good predictor of geometric ability. Duranovic, Dedeic and Gavric (2015) found that children with dyslexia performed equivalently to those without dyslexia on the mental rotations test (Vandenberg & Kuse 1978), non-analytical spatial visualization task. Obviously, children with specific learning difficulties have different development of their spatial visualisation skills. While one group do not have problem with ability to mentally rotate objects and this ability can be used to encourage other problematic skills, the other group have problems and should work directly on development of these skills.

According to van Hiele's theory of geometry mental development levels (van Hiele, 1959), recognition or visualization is the first and most fundamental developmental step for children to learn geometry skills and concepts. Results of the study showed that more than half of the children with geometric difficulties did not have problem with this step, because they are on the second level, which gives us a positive view and the hope they can build these skills. Building on visualization and representation, children develop further levels of geometric reasoning, including analysis (e.g., thinking in terms of properties of visual figures), informal deductive (i.e., noticing relationships within and between figures), formal deduction (i.e., geometry proofs), and rigor (i.e., thinking in terms of abstract mathematical systems) (Gal & Linchevski, 2010).

Geometry skills at level 3 (abstraction/informal deduction) mean that the students master geometry skills at level 2 (analysis) and level 1 (visualisation) (Astuti, Suryadi, & Turmudi, 2018). Most students without geometric difficulties (81.8 %) were on the third level of geometry mental development, and 9.1% of them were on the fourth and on the fifth level. In the study conducted by Astuti, Suryadi, and Turmudi (2018) 23.7% students were at the level 1 (visualization), 44.7% at the level 2 (analysis), and 31.6% were categorized as level 3 (informal deduction). De Villiers and Njisane (1987) noted that 45 % students were at the level 2 or lower. Similar low levels in students in high schools were found by Malan (1986) and Smith and De Villiers (1989). All these results are worse than the results obtained in this study, which can be explained with the fact that they included all students, without making a distinction between students with geometric difficulties or not. Given that 23.7% of students were at the level 1 in the first study, and 45 % students were at the level 2 or lower, there is a real possibility that among examined students there were students with geometric difficulties.

It is important to know the geometry mental development levels of student. Before teaching students for a higher level, it is necessary to examine at what level they are. If a student gets a task that is at a higher level than the student's competencies, the communication will be poor. Such a procedure will create a problem for the students, they will be forced just to memorize and thus only achieve temporary and superficial success (Čižmešija et al., 2010). Once it has been determined to which Van Hiele level the students belong, certain activities can be done with them according to the child's level (Karjaković, 2014).

This study showed a large percentage of children with geometric difficulties in elementary schools. Different research has shown that many of these mathematical difficulties may be connected to problems in the visuospatial domain (Heathcote, 1994; Cornoldietal, 1999; Mammarella et al., 2006, 2010). This study showed problems in this aspect for children with geometric difficulties. Future research should include all children with mathematical difficulties and examining the relationship between deficits in different domains (i.e., deficits in arithmetic and deficits in geometry) in order to better understand their differences and similarities, and to analyze whether children experiencing a deficit in one domain show deficits in the other domain as well. Although comorbidity rates between disorders in these two domains are supposed to be high, it would be significant to determine whether deficits in specific domains also occur individually and do they have different causes and therefore require different interventions.

Prevention and early intervention are acknowledged as key components for reducing the impact of any potentially problems in child development (Shonkoff & Meisels, 1990). Another direction for future research could be building interventions based on risk factors to geometrical difficulties, which would enable the implementation of activities before problems worsen or by preventing their onset (see Marmot et al., 2008).

Although this study has gaven important insights into the geometrical difficulties some limitations should be considered for future research. The small sample size is one of limitations which restricted the possibility of generalizing the results. Future studies would need to examine a much larger sample to represent a large student population. Children from third to fifth grade were included in a sample. It will be important to future studies to investigate developmental mechanisms of geometric difficulties longitudinally, starting with younger age groups and including adults.

REFERENCES

American Psychiatric Association. (2013). *Diagnostic and statistical manual of mental disorders* (5th ed.). Washington, DC: American Psychiatric Association.

Astuti, R., Suryadi, D., & Turmudi, T. (2018). Analysis on geometry skills of junior high school students on the concept congruence based on Van Hiele's geometric thinking level. *Journal of Physics Conference Series, 1132*(1), 012036.

Attneave, F., & Arnoult, M. D. (1956). The quantitative study of shape and pattern perception. *Psychological Bulletin, 53*(6), 452–471.

Baddeley, A. D. (2000). The episodic buffer: a new component of working memory? *Trends in Cognitive Sciences, 4*(11), 417–423.

Badian, N. A. (1983). Arithmetic and nonverbal learning. In H.R., Myklebust (ed) *Progress in Learning Disabilities, Vol 5* (pp 235-264). New York: Grune and Stratton.

Badian, N. A. (1999). Persistent arithmetic, reading, or arithmetic and reading disability. *Annals of Dyslexia, 49*, 45–70.

Battista, M. T. (2002). Learning geometry in a dynamic computer environment. *Teaching Children*

Mathematics, 8(6), 333–339.

Bilbija, D., Milanković, J., & Runjić, N. (2009). *Teorija van Hiele-ovih o razumijevanju geometrije (Van Hiele's theory of understanding geometry).* Banja Luka: Faculty of Science.

Bull, R., Johnston, R. S., & Roy, J. A. (1999). Exploring the roles of the visual-spatial sketch pad and central executive in children's arithmetical skills: Views from cognition and developmental neuropsychology. *Developmental Neuropsychology, 15*, 421-442.

Burger, W. R., & Shaughnessy, J. M. (1986). Characterizing the van Hiele levels of development in geometry. *Journal for Research in Mathematics Education, 17*(1), 31-48.

Butterworth, B., Varma, S., & Laurillard, D. (2011). Dyscalculia: From brain to education. Science*, 332*(6033), 1049–1053.

Bynner, J., & Parsons, S. (2006). *Does Numeracy Matter More? National Research and Development Centre for Adult Literacy and Numeracy.* London: The Basic Skills Agency. Retrieved from http://www.nrdc.org.uk

Cai, D., Li, Q. W., & Deng, C. P. (2013). Cognitive processing characteristics of 6th to 8th grade Chinese students with mathematics learning disability: relationships among working memory, pass processes, and processing speed. *Learning and Individual Differences, 27*, 120–127.

Cass, M., Cates, D., Smith, M., & Jackson, C. (2003). Effects of manipulative instruction on solving area and perimeter problems by students with learning disabilities. *Learning Disabilities Research & Practice, 18,* 112–120.

Chew, C. M., & Lim, C. S. (2013). Enhancing primary pupils' geometric thinking through phase-based instruction using the Geometer's Sketchpad. *Asia Pacific Journal of Educators and Education, 28*, 33-51.

Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. In D. A., Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 420–464). New York: MacMillan.

Collin, C. A., & McMullen, P. A. (2002). Using MATLAB to generate families of similar Attneave shapes. *Behavior Research Methods, 34*(1), 55-68.

Cornoldi, C., Rigoni, F., Tressoldi, P. E., & Vio, C. (1999). Imagery deficits in nonverbal learning disabilities. *Journal of Learning Disability, 32*, 48–57.

Crnjac, D. (2013). *Teorije učenja matematike (Theories of learning mathematics).* Graduation thesis. Osijek: University of J.J. Strossmayera.

Daneman, M., & Carpenter, P. A. (1980). Individual differences in working memory and reading. *Journal of Verbal Learning and Verbal Behavior, 19*(4), 450–466.

Dehaene, S., & Cohen, L. (1997). Cerebral Pathways for Calculation: Double Dissociation between Rote Verbal and Quantitative Knowledge of Arithmetic. *Cortex, 33*(2), 219–250.

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., & Tsivkin, S. (1999). Sources of mathematical thinking: Behavioral and brain imaging evidence. *Science, 284*, 970–974.

De Villiers, M. D. (2008). Uloga i funkcija dokaza u matematici (The role and function of proofs in mathematics). *Poučak, 35*, 13-17.

De Villiers, M. D., & Njisane, R. M. (1987). The Development of Geometric Thinking among Black High School Pupils in KwaZulu. *Proceedings of the Eleventh PME-conference. Montreal: 3*, 117-123.

De Risi, V. (2015). *Mathematizing Space: The Objects of Geometry from Antiquity to the Early Modern Age.* Switzerland: Springer International Publishing.

D'Amico, A., & Guarnera, M. (2005). Exploring working memory in children with low arithmetical achievement. *Learning and Individual Differences, 15,* 189–202.

Duranovic, M., Dedeic, M., & Gavric, M. (2015). Dyslexia and Visual-Spatial Talents. *Current Psychology, 34*(2), 207-222.

Ekstrom, R. B., French, J. W., Harman, H. H., & Dermen, D. (1976). *Kit of factor referenced cognitive tests.* Princeton: Educational Testing Service.

Fuys, D., Geddes, D., & Tischler. R. (1988). *The van Hiele model of thinking in geometry among adolescents. JRME Monograph No. 3*, Washington, D.C.: National Council of Teachers of Mathematics.

Gal, H., & Linchevski, L. (2010). To see or not to see: Analyzing difficulties in geometry from the perspective of visual perception. *Educational Studies in Mathematics, 74*(2), 163–183.

Geary, D. C. (1993). Mathematical disabilities: cognitive, neuropsychological, and genetic components. *Psychological Bulletin, 114*, 345–362.

Geary, D. C. (1996). Sexual selection and sex differences in mathematical abilities. *Behavioral and Brain Sciences, 19*, 229–284.

Geary, D. C., & Hoard, M. K. (2001). Numerical and arithmetical deficits in learning-disabled children: Relation to dyscalculia and dyslexia. *Aphasiology, 15*(7), 635–647.

Geary, D. C. (2003). Learning disabilities in arithmetic: Problem solving differences and cognitive deficits. In H. L. Swanson, K. Harris, & S. Graham (Eds.), *Handbook of learning disabilities* (pp. 199–212). New York, NY: Guilford Press.

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities, 37*, 4–15.

Giofrè, D., Mammarella, I. C., & Cornoldi, C. (2013). The structure of working memory and how it relates to intelligence in children. *Intelligence, 41*(5), 396–406.

Giofrè, D., Mammarella, I. C., Ronconi, L. & Cornoldi, C. (2013). Visuospatial working memory in intuitive geometry, and in academic achievement in geometry. *Learning and Individual Differences, 23*, 114–122.

Gross-Tsur, V., Manor, O., & Shalev, R. S. (1996). Developmental dyscalculia: Prevalence and demographic features. *Developmental Medicine and Child Neurology, 38*(1), 25–33.

Hammill, D. D. (1990). On defining learning disabilities: an emerging consensus. *Journal of Learning Disabilities, 23*, 76-84.

Heathcote, D. (1994). The role of visuo-spatial working memory in the mental addition of multi-digit addends. *Cahiers De Psychologie Cognitive-current Psychology of Cognition, 13*, 207–245.

Hoffer, A. (1981). Geometry is more than proof. *Mathematics Teacher, 74*, 11-18.

Jeung, H., Chandler, P., & Sweller, J. (1997). The role of visual indicators in dual sensory mode instruction. *Educational Psychology, 17*, 329–343.

Karagiannakis, G., Baccaglini-Frank, A., & Papadatos, Y. (2014). Mathematical learning difficulties subtypes classification. *Frontiers in Human Neuroscience, 8*(57), 1-5.

Kessels, R. P. C., van Zandvoort, M. J. E., Postma, A., Kappelle, L. J., & de Haan, E. H. F. (2000). The Corsi Block-Tapping Task: Standardization and Normative Data. *Applied Neuropsychology, 7*(4), 252–258.

Kinach, B. M. (2012). Fostering Spatial vs. Metric Understanding in Geometry. *Mathematics Teacher, 105*(7), 534–540.

Kosc, L. (1974). Developmental dyscalculia. *Journal of Learning Disabilities, 7*, 46-59.

Kroesbergen, E., & Van Luit, J. (2003). Mathematics interventions for children with special educational needs. *Remedial and Special Education, 24*, 97–114.

Lewis, C., Hitch, G. J., & Walker, P. (1994). The prevalence of specific arithmetic difficulties and specific reading difficulties in 9- to 10-year old boys and girls. *Journal of Child Psychology and Psychiatry, 35,* 283-292.

Luck S. J., & Vogel E. K. (1997). The capacity of visual working memory for features and conjunctions. *Nature, 390*, 279–281.

Maccini, P., Mulcahy, C., & Wilson, M. (2007). A follow-up of mathematics interventions for secondary students with learning disabilities. *Learning Disabilities Research & Practice, 22*, 58–74.

Mackworth, N. H. (1948). The breakdown of vigilance during prolonged visual search. *Quarterly Journal of Experimental Psychology, 1*, 6-21.

Malan, F. R. P. (1986). *Teaching strategies for the transition of partition thinking to hierarchical thinking in the classification of quadrilaterals*. Stellenbosch: University of Stellenbosch, Research Unit of Mathematics Education.

Mammarella, I. C., Cornoldi, C., Pazzaglia, F., Toso, C., Grimoldi, M., & Vio, C. (2006). Evidence for a double dissociation between spatial-simultaneous and spatial-sequential working memory in visuospatial (nonverbal) learning disabled children. *Brain Cognition, 62*, 58–67.

Mammarella, I. C., Lucangeli, D., & Cornoldi, C. (2010). Spatial working memory and arithmetic deficits in children with nonverbal learning difficulties. *Journal of Learning Disability, 43*, 455 –468.

Marmot, M., Friel, S., Bell, R., Houweling, T. A., Taylor, S., Health, CoSDo. (2008). Closing the gap in a generation: health equity through action on the social determinants of health. *Lancet,* 372, 9650, 1661–9.

Mazzocco, M. M., Feigenson, L., & Halberda, J. (2011). Impaired acuity of the approximate number system underlies mathematical learning disability (dyscalculia). *Child Development, 82*(4), 1224–1237.

McCloskey, M. (1992). Cognitive mechanisms in numerical processing: Evidence from acquired dyscalculia. *Cognition, 44*(1), 107–157.

McLean, J. F., & Hitch, G. J. (1999). Working memory impairments in children with specific arithmetic learning difficulties. *Journal of Experimental Child Psychology, 74*, 240–260.

Mistretta, R. (2000). Enhancing geometric reasoning. *Adolescence, 35*, 369–379.

Morsanyi, K., van Bers, B. M. C. W., McCormack, T., & McGourty, J. (2018). The prevalence of specific learning disorder in mathematics and comorbidity with other developmental disorders in primary school-age children. *British Journal of Psychology, 109*(4), 917-940.

Mueller, S. T. (2010). A partial implementation of the BICA Cognitive Decathlon using the Psychology Experiment Building Language (PEBL). International *Journal of Machine Consciousness, 2*, 273–288.

Mueller, S. T. & Esposito, A. G. (2014). Computerized Testing Software for Assessing Interference Suppression in Children and Adults: The Bivalent Shape Task (BST). Journal *of Open Research Software, 2*(1), e3.

Mueller, S. T. & Piper, B. J. (2014). The psychology experiment building language (PEBL) and PEBL test battery. *Journal of Neuroscience Methods, 222*, 250-259.

National Council of Teachers of Mathematics. (NCTM). (2000). Principles *and standards for school mathematics*. Reston, VA: Author.

National Mathematics Advisory Panel. (2008). *Foundation for success: The final report of the National Mathematics Advisory Panel.* Washington, DC: U.S. Department of Education.

National Research Council (NRC). (2001). Adding it up: Helping children learn mathematics. In J. Kilpatrick, J. Swafford, & B. Findell, (Eds.), *Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education*. Washington, DC: National Academy Press.

Ovčar, S. (1990). *Razvoj mišljenja u nastavi matematike (Development of thinking in teaching of*

mathematics). Čakovec: Zrinski.
Passolunghi, M. C., & Pazzaglia, F. (2004). Individual differences in memory updating in relation to arithmetic problem solving. *Learning and Individual Differences, 14*(4), 219–230.
Pedemonte, B., Watson, C., Borghesani, V., Ebbert, M., Allen, I. E., Pinheiro-Chagas, P., De Leon, J., Miller, Z. , Tee, B. L., & Gorno-Tempini, M. L. (2022). A novel approach to subtypes of developmental dyscalculia. *Research Square*. Retrieved: file:///C:/Users/KorisnikPC5/Downloads/A_novel_approach_to_subtypes_of_developmental_dysc-1.pdf
Raven, M. S. (1956). *Uputstvo za korišćenje Progresivnih matrica u boji (Instructions for the use of colored progressive matrices)*. Beograd: Centar za primenjenu psihologiju.
Razel, M., & Eylon, B. S. (1991). *Developing mathematics readiness in young children with the Agam Program*. Proceedings of the Fifteenth Conference of the International Group for the Psychology of Mathematics Education. Genova, Italy.
Rourke, B. P., & Conway, J. A. (1997). Disabilities of arithmetic and mathematical reasoning: perspectives from neurology and neuropsychology. *Journal of Learning Disabilities, 30*, 34–46.
Rubinsten, O., & Henik, A. (2009). Developmental Dyscalculia: Heterogeneity might not mean different mechanisms. *Trends in Cognitive Sciences, 13*(2), 92–99.
Shalev, R. S., Auerbach, J., Manor, O., & Gross-Tsur, V. (2000). Developmental dyscalculia: prevalence and prognosis. *European Child & Adolescent Psychiatry, 2*, II58-64.
Shalev, R. S. (2007). Prevalence of developmental dyscalculia. In D. B. Berch & M. M. M. Mazzocco (Eds.), *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities* (p. 49–60). US: Paul H. Brookes Publishing Co.
Shepard, R. N. & Metzler, J. (1971). Mental rotation of three-dimensional objects. *Science, 171*, 701 –703.
Shonkoff, J. P. & Meisels, S. J. (1990). Early childhood intervention: The evolution of a concept. In: S. J., Meisels, & J. P., Shonkoff, eds. *Handbook of early childhood intervention*. New York: Cambridge University Press; p. 3–31.
Simic, N., Khan, S., & Rovet, J. (2013). Visuospatial, visuoperceptual, and visuoconstructive abilities in congenital hypothyroidism. *Journal of the International Neuropsychological Society, 19* (10), 1119–1127.
Smith, E., & De Villiers, M. (1989). *A comparative study of two Van Hiele testing instruments*. Poster presented at the 13International Conference of PME, Paris.
Šutalo, Ž. (2016). *Ključne ideje u podučavanju matematike (Key ideas in teaching of mathematics).* Graduation thesis. Osijek: University of J. J. Strossmayera.
Triadafillidis, T. A. (1995). Circumventing visual limitations in teaching the geometry of shapes. *Educational Studies in Mathematics, 29*(3), 225–235.
Usiskin, Z. P. (1982). *Van Hiele levels and achievement in secondary school geometry*. Chicago, IL: University of Chicago, Department of Education.
Vandenberg, S. G., & Kuse, A. R. (1978). Mental rotations, a group test of three-dimensional spatial visualization. *Perceptual and Motor Skills, 47*, 599–601.
Van Hiele, P. (1959). *The child's thought and geometry*. Brooklyn: City University of New York.
Van Hiele, P. (1986). *Structure and insight: A theory of mathematics education*. Orlando, FL: Academic Press.
Venneri, A., Cornoldi, C., & Garuti, M. (2003). Arithmetic difficulties in children with visuospatial learning disability (VLD). *Child Neuropsychology, 9*, 175–183.
Volenec, V. (1979). Geometrija: Tehnička enciklopedija (Geometry: Technical Encyclopedia). Zagreb: Leksikografski zavod Miroslav Krleža.

Von Aster, M. G., & Shalev, R. S. (2007). Number development and developmental dyscalculia. *Developmental Medicine & Child Neurology, 49*(11), 868–873.

Zhang, D., Ding, Y., Stegall, J., & Mo, L. (2012). The Effect of Visual-Chunking-Representation Accommodation on Geometry Testing for Students with Math Disabilities. *Learning Disabilities Research & Practice, 27*(4), 167–177.

Wadlington, E., Wadlington, P. (2008). Helping students with mathematical difficulties to succeed. *Preventing School Failures, 53*(1), 2-7.

Wong, W. -K., Hsu, S. -C., Wu, S. -H., Lee, C. -W., & Hsu, W. -L. (2007). LIMG: Learner-initiating instruction model based on cognitive knowledge for geometry word problem comprehension. *Computers & Education, 48*, 582–601.