Asia Pacific Journal of Developmental Differences
Vol. 10, No. 1, January 2023, pp. 27—63
DOI: 10.3850/S2345734123000274

The experiences of Primary 6 students with dyslexia using the metacognitive-based approach of problem solving for algebraic word problems

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Abstract

Research has shown that students with dyslexia internationally can struggle with aspects of Mathematics. Moreover, they specifically struggle with word problems, because of the mathematical language and multi-steps involved, and the demands on working memory. At the Dyslexia Association of Singapore (DAS), the Problem Sums for Upper Primary (PSUP) curriculum was developed in 2016 to meet the needs of our primary school students who were firm in their understanding of basic mathematical concepts but lacked the appropriate strategies to solve higher-order word problems. The PSUP curriculum utilizes a combination of Polya's 4-step processes, the Concrete-Representational-Abstract approach (C-R-A) and the Try-Share-Learn-Apply approach as its primary teaching methodology. As the programme has yet to explore students' meta-cognitive abilities in planning, monitoring, solving and checking word problems, this study aims to understand the thought processes of eight Primary six students in solving word problems involving Algebra concepts through interviews and a series of tests. To evaluate the effectiveness of the PSUP curriculum teaching approaches, the students' pre-test, review test and post-test scores were compared. The results showed that 75% of the students improved from Pre-test to Review test and from Pre-test to Post test for all the algebraic concepts taught. Students were given a questionnaire at the beginning and end of the intervention period to assess their confidence level in solving Mathematics problem sums. Responses from the questionnaires also showed that the students were more confident in solving word problems as compared to at the start of intervention. Limitations and instructional implications will also be discussed. Further research into the students' meta-cognition before and after solving word problems would give a deeper insight to how their thought processes may have evolved, and how the use of our structured metacognitive-based approach has an impact on them.

Keywords: Mathematical language and multi steps. Metacognitive-based approach. planning, monitoring, solving and checking word problems

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INTRODUCTION

Dyslexia, Maths and Metacognition

Dyslexia is a specific learning difference that makes it difficult for people to read, write and spell (DAS, 2022b). Apart from English, the significance of maths in daily life is something that cannot be overstated, including for example, the ability to work out the discount for items they want to purchase. The development of the applications that we use and even aspects of engineering involve the use of mathematics at various levels. As such, it is important for students to be able to comprehend mathematical concepts and solve questions. In terms of mathematical word problem sums, metacognition is an important factor.

Relationship between Metacognition and Maths

Metacognition in essence is the awareness and understanding on one's own thought processes. Developing metacognition is important as it can improve an individual's ability to apply knowledge and skills in areas outside the context of which they were learned. This ability to transfer knowledge and skills across various areas of life is vital in general. Bringing the context back to mathematics, when working on word problem sums, students need to be aware of what they are doing, why they are doing it and be able to regulate these processes and adjust their plans if necessary.

How Dyslexia Impacts an Individual's Math Ability

However, Dyslexia could affect an individual's mathematical ability as well. There has been consistent evidence for difficulties with Mathematics in dyslexia, with a recent study comparing 193 adult dyslexics with controls showing that 62% had problems with subtraction with no evidence for difficulties in the controls (Reynolds and Carovalas, 2016). Students with dyslexia have certain inherent weaknesses which makes mathematics challenging for them, for example reversing numbers when writing. They may struggle with directionality as problems can be solved from right to left, top to bottom or even left to right, depending on the question. Weak working memory could lead to challenges in simple mathematical computations, for example, the ability of students to memorise the times tables (with 93% dyslexic adults impaired in tables in Reynolds and Carovalas, 2016). Even their execution of multi-step questions, which would require proper sequencing, can be impacted, as well, with students consistently losing their place and missing out stages in their working. Students may also face organisational and planning difficulties such as an inability to come up with a plan and strategize in order to determine a solution. Slower processing may lead to difficulties with abstracting information as well. Students with dyslexia could face difficulties with their language comprehension and reading skills as they may struggle to read the questions accurately and misunderstand the questions. Impulsivity and lack of inhibition, associated with ADHD which is heavily co-morbid with dyslexia, may also lead to inability to check and reflect on their own thinking. For an overview of the difficulties encountered in Mathematics for children with dyslexia, see Chinn, (2015).

Evidence for the type of difficulty encountered, and the overlap between reading and Mathematics disabilities has been widely researched internationally. In 1999, Badian in the USA in a major study of over 1000 children found that 3.4% showed difficulties in both reading and Mathematics. In a series of family studies, Landerl and Moll, (2010) established that reading and Mathematics deficits were significantly transmitted together, with Joyner and Wagner (2019) showing that students with Mathematics disability had more than twice the incidence of Reading disability. Most recently, a high-risk family study from Snowling and colleagues in the UK (Snowling, Moll and Hulme, 2021) found that 60% of children with RD had MD as well. These findings clearly establish the importance of Mathematics support for many children with dyslexia.

In terms of learning Mathematics, both declarative (number facts) and procedural (algorithms for performing a calculation) are needed. These need to work in tandem in order to achieve success. On a theoretical level, both Procedural learning and the process of consolidation (the final stage in learning) have been shown to be impaired in children with dyslexia (Nicolson and Fawcett, 2007). More repetition is needed for students with dyslexia to firmly acquire concepts, and their skills are likely to remain more fragile under stress.

Moreover, these is a clear additive role here, for the impact of anxiety in Mathematics on dyslexic students' performance, with evidence for this from research internationally.

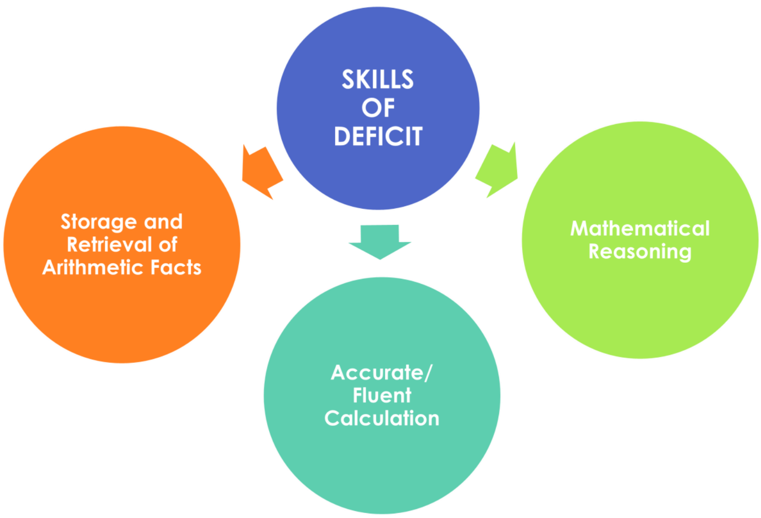


Figure 1. Learning Difficulties in Mathematics (DSM-5 diagnostic code 315.1)

Based on DSM-5 (Diagnostic Statistical Manual of Mental Disorders, 5th edition), dyslexia, which is now subsumed under the term "specific learning disorder" under diagnostic code 315.1., can be accompanied with an "impairment in mathematics".

Having such a diagnosis indicates that a child shows possible deficits in:

1. Number sense
2. Memorisation of arithmetic facts
3. Accurate or fluent calculation
4. Accurate math reasoning

The Role of Mathematics Anxiety in Dyslexia.

Interestingly, Mathematics anxiety is present even at the beginning of formal schooling (Maloney, 2012) and many of the techniques used to address this are based on addressing the phobias rather than the Mathematics itself. A systematic review (Mugnaini et al., 2009) has shown that dyslexia is a specific risk factor for anxiety and other internalising disorders. The stress associated with failure creates a vicious cycle of further failure. In a questionnaire study in 2008, Chinn demonstrated the role of the environment and personality as well as intellectual strengths or difficulties for dyslexia and Mathematics. Notably, secondary level dyslexic students tended to give no response answers based on their lack of confidence in Mathematics, with working memory issues exacerbating their difficulties.

As such, intervention in mathematics is important to help dyslexic students become more successful mathematics learners as well as build their confidence in their mathematical abilities. The topic on which this article focuses, problem solving, encapsulates many of the difficulties that dyslexic children experience in terms of metacognitive skills and strategy use. Problem solving demands the ability to read fluently, maintain the steps in working memory, follow the sequence through in the correct order, and deal with negatives or contradictory instructions, in order to reach the correct answer. For many dyslexic children, word problems highlight their difficulties in speed and accuracy, and thus they require a systematic approach with a formula that can help to build their metacognitive skills and reduce their anxiety in dealing with the word problems that can form the majority of the syllabus for this age group.

The Mathematics Curriculum and Problem Solving in Singapore

In Singapore problem solving has become central to the mathematics curriculum since 1992, (Lee 2008). More than 60% of the Mathematics curriculum for Primary schools constitutes solving word problems. Mathematical problem solving alludes to the importance of having students know how to form meaningful relationships from the given information in the word problem and to relate the relationships to the mathematical operations (Van de Walle, 2004).

Word Problem sums generally require students to be able to accurately comprehend and organise the information provided, come up with a plan to solve, sequence the operations and steps used, and if the questions requires two or more steps, write number sentences, complete workings and adjust their plan if necessary. As such, being able to not only comprehend but solve word problem sums is vital for all students in general.

The inseparable link between assessment and curriculum points to the fact that word problems are a mainstay in the Singapore primary mathematics curriculum and has led many schools to teach problem solving by teaching word problems. The word problems include non-routine, open-ended and real world problems.

The development of Mathematical problem solving ability is dependent on five interrelated components, namely, concepts, skills, processes, attitudes and metacognition. Each time a new concept is introduced, problem solving becomes central.

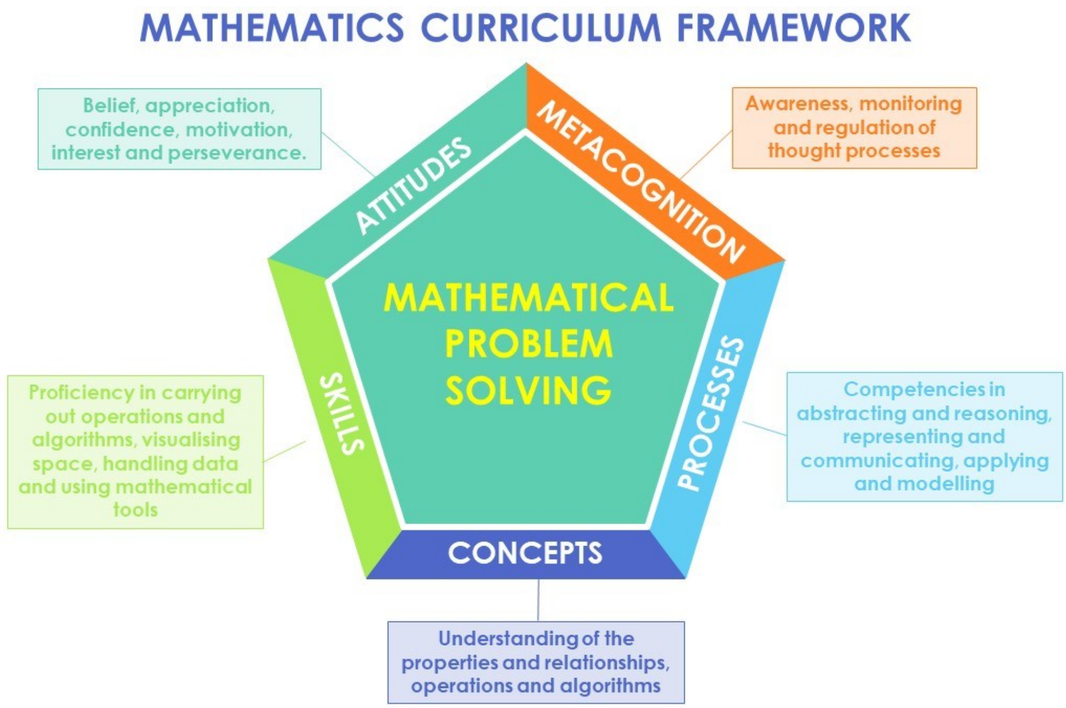


Figure 2. . Components of mathematical problem solving.

Mathematics Programmes at the Dyslexia Association of Singapore

The Mathematics Programme at the Dyslexia Association of Singapore has been designed to effectively support students with dyslexia who have persistent difficulties with mathematics. As students may have diverse mathematical ability, the program currently offers three main curricula; The Essential Maths Programme, Secondary 1 Normal Technical Programme and the Problem Sums for Upper Primary Programme also known as PSUP (DAS, 2022). In this article, the PSUP will be evaluated for effectiveness.

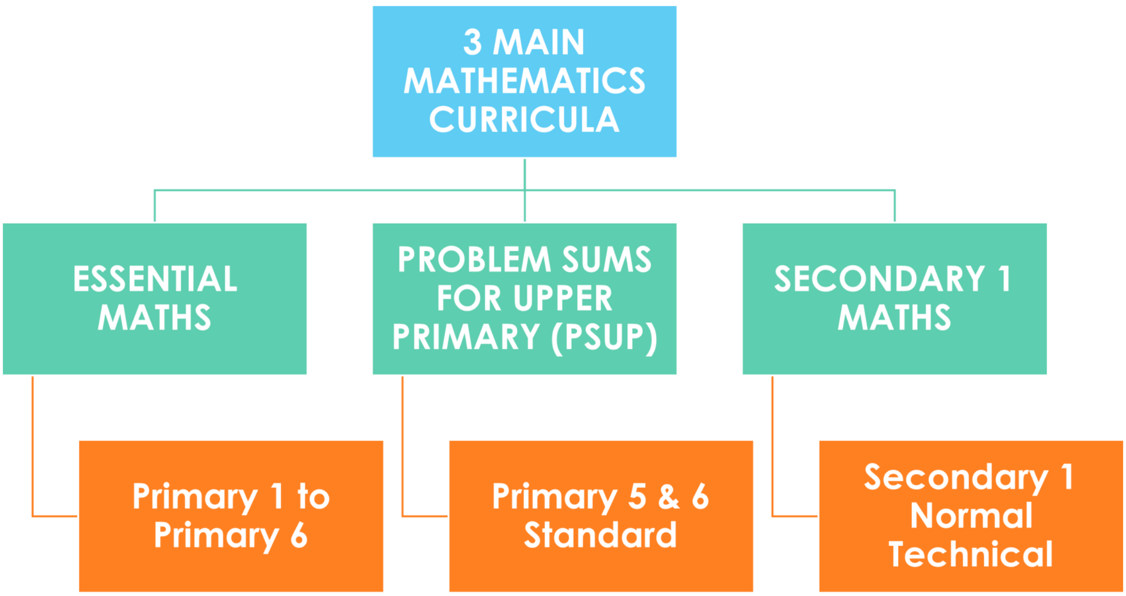


Figure 3. The Mathematics curricula at DAS, Singapore

Components of the Problem Sums for Upper Primary Programme (PSUP)

The Problem Sums for Upper Primary Programme was developed in 2016 to address the needs of students in Primary 5 and 6 Standard who were firm with their basic mathematical concepts but needed more support to understand and solve word problem sums. The programme focuses on teaching students to identify challenging word problem types in each topic as well as how to use reading comprehension skills and heuristic strategies to comprehend and solve those problems.

The programme as a whole utilizes various teaching approaches such as the Orton Gillingham Approach (where classes are language based, cognitive and diagnostic and prescriptive. Lessons are structured, cumulative and sequential as well as emotionally sound. Direct instructions are given to students as well.

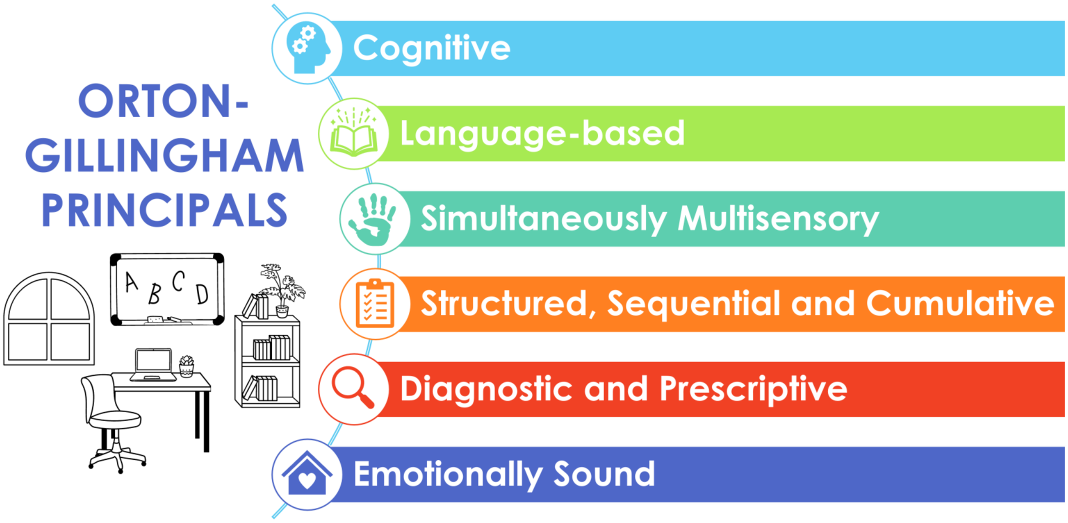


Figure 4. Orton Gillingham principles used in DAS

The Problem Sums for Upper Primary Programme utilizes a combination of other approaches such as Po'lya's 4 step processes to help student understand the problem, come up with a plan to solve the problem, solve the problem and check the solution.

Figure 5. Po'lya's 4 step process (Po'lya, 1971)

To augment the teaching of problem solving, mathematics teachers in Singapore mainstream schools and at the Dyslexia Association of Singapore train students on Po'lya's (1971) approach towards problem solving (Ministry of Education, 2006).

Po'lya (1971) described mathematical problem solving as comprising four stages, namely, understanding, planning, carrying out the plan, and looking back. This model encourages students to first read and understand the problem, particularly identifying the information given and the information that needs to be found. Students then proceed to devise a plan by determining the methods, strategies or heuristics that are applicable and to decide on the most appropriate or effective approach. Subsequently, the model guides students to carry out their plan to solve the problem and to switch strategies if the planned approach did not work. Finally, the model prompts students to look back at their solution to determine if their answer is reasonable within the problem context, and that they have indeed solved the problem by finding what they are required to find. Students are also to reflect about the problem-solving process and determine which part of their solution worked and which did not so as to train themselves to predict the strategy that could be used to solve other similar problems in future.

Thus the problem-solving process is modelled in classroom lessons, and students are guided to successfully solve the word problems applying recommended heuristics introduced to them by their mathematics teachers.

In addition, in integrating problem solving in the classroom, the Mathematics teacher creates an environment in which students discover more than one approach to solve given problems. Such an environment is conducive in promoting learning for all students and supports students with different learning styles. This view is supported by Moser (1992) who reasons that "an orientation towards problem solving can accommodate individual differences, especially if the philosophy is adopted that there is more than one way to solve most problems.

The Concrete-Representational and Abstract approach

Students are given opportunities to use physical objects to experience the concept in real life, before the objects are represented using pictures so as to help students reduce their dependence on the physical manipulatives to solve the question. At the abstract stage, students can draw on their understanding of the problem as shown to them during the Concrete and Representational Stage to solve the questions.

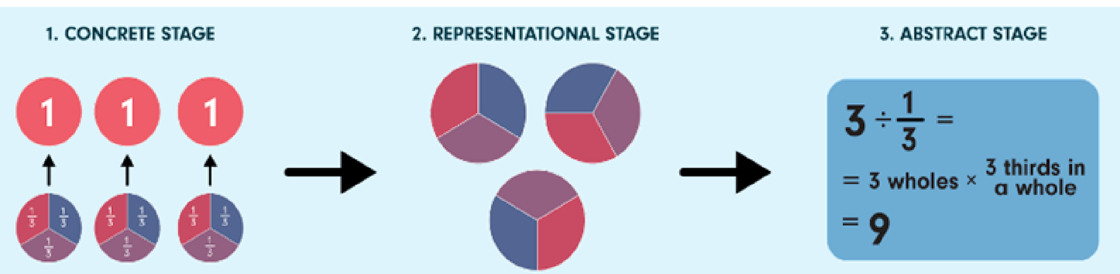


Figure 6. From concrete representational to abstract stages (DAS, 2022a)

Lastly, a try-share-learn-apply approach is used as well to help students walk through the problem-solving process in a collaborative and interactive setting.

Figure 7. Collaborative and interactive learning

To enable students to solve problems effectively, students need to be equipped with effective problem solving strategies so that they can apply them in problem solving. To achieve this end, a number of heuristics are explicitly introduced to the students when a new concept is introduced to them to enhance their problem solving performance.

METHODOLOGY

The type of research that was used in this case study is qualitative and quantitative in nature. Results of pre, mid-post (review) and post-tests of several mathematical topics were analyzed and students were given questionnaires before and after the intervention to express their confidence levels. They were also interviewed on the processes of their problem-solving skills and techniques used.

Participants

The mean ages of the children are indicated in the table below:

Table 1. Mean ages of children participating

	Girls	Boys
Mean Age	13.41	13.44
Range	12.91 - 13.78	12.96 – 14.02

The reason students are enrolled in the PSUP programme is to improve on their problem-solving skills by learning appropriate strategies for the different problem types. These students were in the Essential Mathematics Programme before coming on PSUP as they have shown the potential to take on higher order Mathematics.

A total of 8 primary six students- 4 boys and 4 girls, all 12 years of age participated in this case study. All the students had previously received a diagnosis of dyslexia from a psychologist and have learning difficulties in Language and also Mathematics. The students involved in the research were from two different centres and taught by two different teachers. Two students were from Centre (A) and were taught by Teacher A. The other six students were from Centre (B) and taught by Teacher B. All the eight students were taking P6 Standard Mathematics in their mainstream school. This means that they are firm in their understanding of their basic Mathematics concepts. Based on our in-house profiling assessment, the students are of average ability. Students in the Essential Mathematics Programme are mostly Average to Low support learners. Upon registering for the DAS Math programme, all students undergo an in-house profiling test, which the Maths Team has developed, based on their respective school levels and streams. The test evaluates number sense and the four operations of Whole numbers. Fractions are included only at the Primary 6 level. The profiling assessment results are analysed and students of similar ability are placed together to form classes based on their current Mathematics abilities and school level. $\frac{7}{8}$ of the students on the Problem Solving for Upper Primary (PSUP) were recommended by their Mathematics Educational Therapists (Edt) while 1 student applied to join the PSUP programme.

Procedures

A case study was conducted over a period of 10 weeks in Term 1 2021 from Week 1 to Week 10. Ten concepts are covered in each term and each concept covers two hours.

Table 2. Concepts covered by the case study group in Term 1 2021

Week	Topic
1	Algebra
2	Simultaneous concept
3	Simultaneous concept
4	Division of Fractions
5	Equal concept involving ratio
6	Excess and Shortage
7	Penalty Charge
8	Ratio – Quantity and Value
9	Percentage – Increase and Decrease
10	Review & Post test (first hour); PSUP Research Student Interviews (2nd hour)

A key feature of the approach was the development of structured worksheets, that lead the students through the processes needed systematically, providing background information on the task, and a system for checking their progress. See appendix 3 – Student Worksheet structure

For the purpose of this research, Algebra and Simultaneous concepts were selected from the Primary 6 Mathematics syllabus. The topic of Algebra was chosen as it was new for the students and it required them to use letters to represent unknown quantities and express relationships, which are abstract for them.

The algebra concept is the first concept of the PSUP curriculum in Week 1 followed by the Simultaneous concept which is an extension of the Algebra concept in Week 2 of the term.

To start on the P6 PSUP curriculum, students first did a one question 5-minute pre-test on Algebra before they were formally introduced to the algebra concept. The recommended strategy for algebra concept is 'Draw a part-whole model' applying the PSUP in-house 'Try Share Learn Apply' approach. At first, at the Try stage, students worked out a question posed to them. At the Share stage, each student shared his/her solution strategy. Then at the Learn stage, Teachers introduced a recommended strategy. Students practiced the recommended strategy with about five questions and then brought back a question as homework practice. At the start of the next lesson in Week 2, students did a one question 5-minute review test to assess how well they understood the algebra concept. They then did a one question 5-minute pre-test on the Simultaneous concept before this concept was next introduced to them.

The recommended strategy for the Simultaneous concept is 'common multiples'. As the Simultaneous concept involved higher order thinking processes, two weeks were allowed for learning this concept. Each week, students practiced with about 2-3 questions and brought back a homework question for reinforcement practice at home. At the start of Week 4, students did a one question 5-minute review test to check their understanding of the Simultaneous concept.

Students undertook a questionnaire to express their confidence levels. They were also interviewed on the processes of their problem-solving skills and techniques used
The algebra concept is the first concept of the PSUP curriculum in Week 1 followed by the Simultaneous concept which is an extension of the Algebra concept in Week 2 of the term.

In the subsequent six weeks, the students learned the following concepts: division of fractions, equal concept involving ratio, excess and shortage, penalty charge, ratio - quantity and value and increase and decrease of percentage.

In the first hour of the 10th week lesson, the teacher reviewed all the eight concepts covered in the term (Table 1) with the students first before letting them do the termly post test on the eight learned concepts. In the second hour, students did a questionnaire (see Appendix 1) to measure their confidence level in doing the post test. An interview questionnaire (see Appendix 2) was also done to find out about their thought processes in learning the algebra and simultaneous concepts.

RESULTS AND FINDINGS

Performance of the participants was compared at pre, review and post-test and the results presented in the figure below.

Pre-test vs Review-test vs Post-tests scores

A comparison of student performance at pre-test, review and post-test is presented in the figures below.

Results for Algebra Concept:

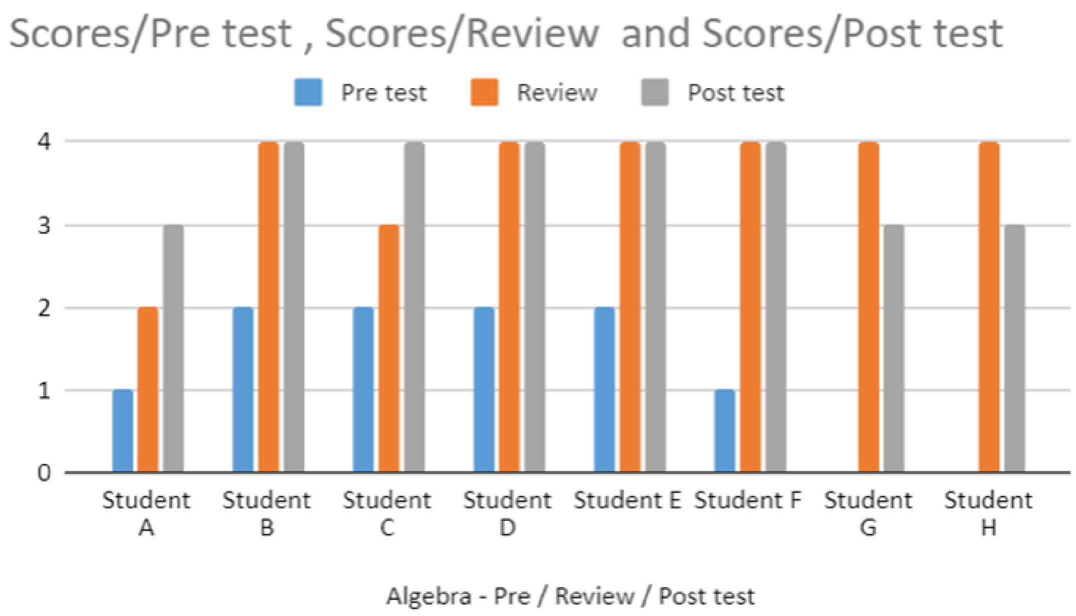


Figure 7. Bar chart of performance on Algebra concept

Each question in the Pre - test, Review test and Post-test carried a maximum of 4 marks.

Table 3. Students' performance at Pre - test (before teaching):

Total of	Obtained	Obtained	Obtained	Obtained	Obtained
Number of	2	2	4	0	0

In the Algebra pre- test, out of the 8 students who participated in the survey, 2 students obtained a zero score, 2 students obtained 1 mark and 4 students obtained 2 marks.

Table 4. Students' performance at Review test (immediate test after teaching):

Total of	Obtained	Obtained	Obtained	Obtained	Obtained
Number of	0	0	1	1	6

However, at the Review test on Algebra in the following week, 1 student obtained 2 marks, 1 other student obtained 3 marks while the majority (6 students) obtained the maximum 4 marks. On the whole, all 8 students demonstrated improved scores.

Table 5. Students' performance at Post-test (delayed test at end of term):

Total of	Obtained	Obtained	Obtained	Obtained	Obtained
Number of	0	0	0	3	5

At end of term Post-test, 3 students obtained 3 marks while the other 5 students obtained the maximum 4 marks.

Thus it was observed that students retained their learning of the algebra concept.

Results for Simultaneous concept

Simultaneous concept- Pre-test, Review test and Post test scores:

Similarly to the algebra concept, each question in the Pre - test, Review test and Post-test of the Simultaneous concept also carried a maximum of 4 marks.

Table 6. Students' performance at Pre - test (before teaching)

Total of	**Obtained**	**Obtained**	**Obtained**	**Obtained**	**Obtained**
Number of	3	2	2	1	0

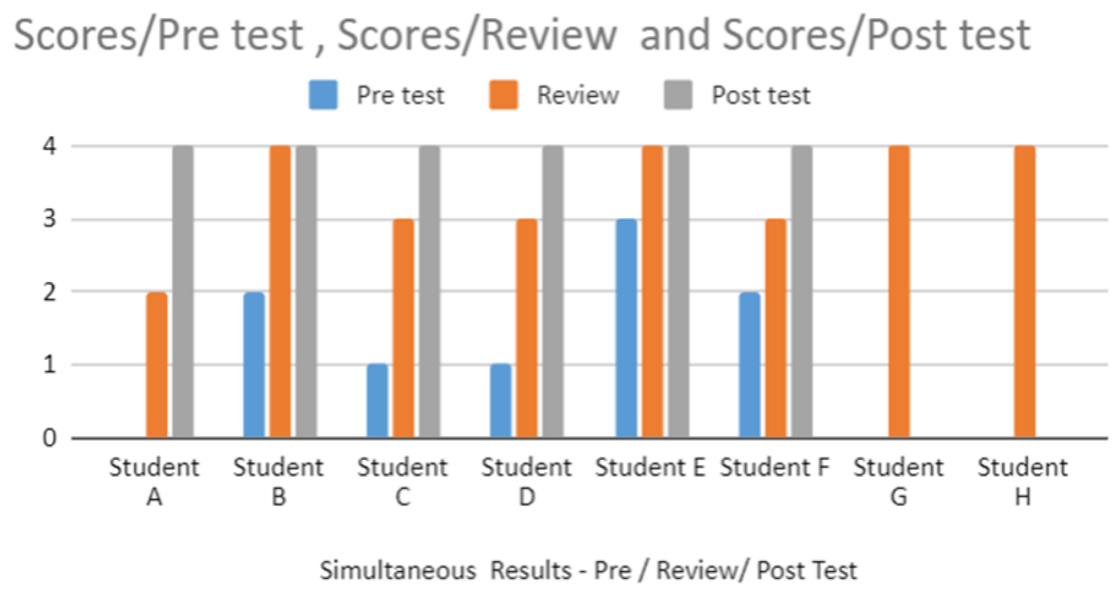


Figure 9. Bar chart of performance on Simultaneous concept

At the pre- test of the Simultaneous concept, out of the 8 students who participated in the survey, 3 students obtained a zero score, 2 students obtained 1 mark, another 2 students obtained 2 marks and only 1 student obtained 3 marks.

Table 7. Students' performance at Review test (immediate test after teaching):

Total of	**Obtained**	**Obtained**	**Obtained**	**Obtained**	**Obtained**
Number of	0	0	1	3	4

However, at the Review test on Simultaneous concept in the following week, 1 student obtained 2 marks, 1 other student obtained 3 marks while the other 4 students obtained the maximum 4 marks. On the whole, all 8 students demonstrated improved scores.

Table 8. Students' performance at Post-test (delayed test at end of term):

Total of	**Obtained**	**Obtained**	**Obtained**	**Obtained**	**Obtained**
Number of	2	0	0	0	6

At the end of the term Post-test, 2 students obtained zero marks while the other 6 students obtained the maximum 4 marks.

Thus, it was observed that 2 of the students who obtained the maximum 4 marks in the review test did not retain their learning of the simultaneous concept and so scored zero marks. They were from Centre (A).

Their reasons for the poor performance were:

- their teachers from mainstream school had not yet taught them the mathematics equations strategy to solve the genre of questions related to Simultaneous concept.
- The equations strategy was also not practiced in working out the solutions for sums in the topics that were taught after so the strategy was forgotten.
- In Centre B the teacher reinforced the equations strategy in the topic of Percentage increase / decrease concept that was taught in Wk 5.
- Although 3 of the students scored a zero in the pre-test, there may have been individual differences in their level of understanding that the pre-test was not sensitive enough to capture.

The results show that 75% of the students successfully understood and managed learning the simultaneous concept while the other 25% regressed in their scores and so need reteaching.

Using of Strategies Taught

In teaching the Algebra concept, the recommended strategy was drawing a part whole model to label and visualise the information from the given sum together with the equation statements.

In the review test on Algebra, the six students from Centre (B) applied the recommended strategy, while the two students from Centre (A) did not. However, at end of term Post test all the students successfully applied the strategy. Table 9 shows the information.

The recommended strategy for teaching the Simultaneous Concept is the writing of Equations to transfer the information in the problem situation into meaningful mathematical equations:

e.g. $4s + 6y = 60$ (Equation 1)
$2s + 3y = 75$ (Equation 2)

Table 9. Strategy use in Algebra

	ALGEBRA—PRE / REVIEW / POST TEST					
	Scores			Recommended Strategy: Draw a part-whole model		
	Pre Test	Review	Post Test	Pre Test	Review	Post Test
Student A	1	2	3		Y	Y
Student B	2	4	4		Y	Y
Student C	2	3	4		Y	Y
Student D	2	4	4		Y	Y
Student E	2	4	4		Y	Y
Student F	1	4	4		Y	Y
Student G	0	4	3		N	Y
Student H	0	4	3		N	Y

Legend

Y = Used the recommended strategy
N = Did not use strategy

Students needed to use the common multiples strategy to make one of the unknowns in the two equations common and then eliminate the common multiple in order to solve the sum.

Students' performance from the review and post test showed that only the 2 students from Centre A did not apply the strategy. The table below shows the findings:

Algebra concept - In the pre-test, Student E and Student F applied the model strategy to work out the solutions. The other students applied the unitary method as the solution. In the review test, students applied the recommended strategy- model drawing. However, in the post test students applied a combination of model drawing and unitary approach to solving the word problems.

Table 10. Strategy use in Simultaneous concept

	SIMULTANEOUS RESULTS—PRE / REVIEW / POST TEST					
	Scores			Recommended Strategy: Common Multiples		
	Pre Test	Review	Post Test	Pre Test	Review	Post Test
Student A	0	2	4		Y	Y
Student B	2	4	4		Y	Y
Student C	1	3	4		Y	Y
Student D	1	3	4		Y	Y
Student E	3	4	4		Y	Y
Student F	2	3	4		Y	Y
Student G	0	4	0		Y	N
Student H	0	4	0		Y	N

Legend

Y = Used the recommended strategy
N = Did not use strategy

Simultaneous concept - All the students worked out the solutions to the questions in the pre/review and post-tests by writing unitary equations relevant to the sums.

In the review test, students applied the strategy introduced to them. However, in the post tests on Algebra and Simultaneous Concept, students applied a combination of both model and unitary approaches to solving the problems.

It is interesting to note that the students who used the unitary method of writing equations to arrive at the solution had already mentally mastered the concepts and wanted to document the workings in the shortest way. This is evident of their increased ability to understand, plan and solve effectively. Thus the success of the PSUP problem solving intervention processes.

Interview Questions on Students' working processes after the Post-test

Table 11. Interview responses

Interview Questions and Student Responses	
1.	**On a scale of 1 to 5, how easy was this test for you? 1 being very easy, 5 being very difficult.**
	3 students - 2; 3 students - 3; 1 student – 1; 1 student no response
2.	**Which question did you find the easiest?**
	Algebra concept Question
3.	**Which question did you find challenging?**
	Simultaneous concept Question
4.	**I would like you to explain to me in detail what went through your mind when you saw question 1 (Algebra Question)**
	When ate 2 means minus 2 then divide; Understand how to do
5.	**What strategies did you use to help you understand this problem?**
	Read meaningfully; Apply Polya processes
6	**Was the strategy you used helpful? If not, what did you do?**
	Yes; Think deeply
7.	**Can you tell me more about the sequence of steps you took to solve this question?**
	Work out the Maths equations following the events in the question
8.	**What happens after you carried out your steps? What did you do to check your work?**
	I double checked by putting the answer number into the question and worked out the sum; I check that the numbers and calculations are correct
9.	**How easy or difficult did you find this question? 1 being very easy, 5 being very difficult**
	5 students – 2; 2 students – 3; 1 student did not answer
10	**Let's move on to Question 2 (simultaneous equation). Can you describe to me in detail how you approached this question?**
	I had to read it multiple times to understand the question. Draw pictures to visualise the situations

Interview Questions and Student Responses (Cont’)	
11	**What strategies did you use to help you understand this problem?**
	I read it repeatedly; I draw pictures to visualise and understand
12	**Was this strategy you used helpful? If not, what did you do?**
	Find how much 1 desk and 1 chair is; Find how much each cost then find the cost of 2 chairs.
13.	**Can you tell me more about the sequence of steps you used to solve this problem?**
	First, I find the cost of 1 chair. Then I can find the cost the desk.
14.	**What happens after you have carried out your steps? What did you do to check your work?**
	I used Work backwards strategy; I make sure that the numbers and calculations are correct.
15.	**How easy or difficult did you find this question? 1 being very easy, 5 being very difficult**
	4 students – 3; 3 students – 5; 1 student - 2
16.	**Let's look at one of the questions you found challenging. Could you tell me why this question is challenging for you?**
	Difficult to understand the meaning
17.	**Can you describe how you approached this question?**
	Reread it meaningfully; Annotate – underline key words, draw boxes
18.	**At which point did you get stuck? What did you do then?**
	Reread it meaningfully; Draw a model; Move on to the nest question
19.	**You have attended our Problem Sums class for 1 term. How do you find the lessons?**
	I draw models, write equations, list the steps, draw pictures to solve the questions; The questions are helpful in examinations; Lessons are fun; Worksheets are good.
20.	**What are some strategies you have picked up from our lessons?**
	Underline keywords; Unitary method; Draw models and diagrams; Write Mathematics equations; Act out the events
21	**(a) How likely will you use these strategies in your school work? 1- not at all, 5- I will definitely use them (b) Tell why you chose this score.**
	(a) 2 students – 3; 2 students – 5; 1 student – 2; 3 students did not answer (b) Useful; Will help in getting method mark
22	**What do you like about the PSUP lessons?**
	Helps to catch up in class (school). Don’t understand in school; Interactive; I can have friends to help me learn.
23	**How can we improve our lessons for future batches of students?**
	Shorter questions

Observations of student responses about the post-test:

Students generally found PSUP lessons fun, interactive and useful. Students could apply the learnt strategies and heuristics with school examination question types.

Questionnaire on Student's Confidence Level in Mathematics

Question - How confident are you in Solving Mathematics Problem sums

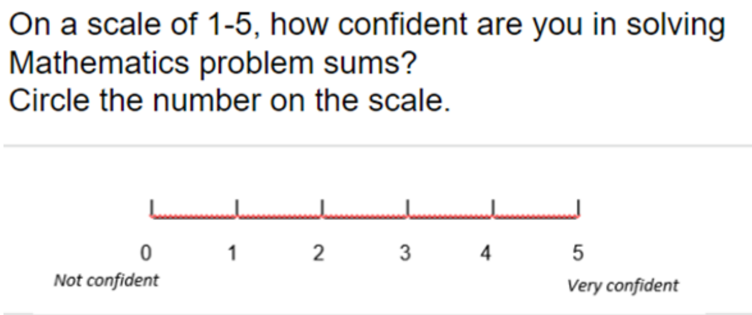


Table 12. Students Confidence level in Mathematics Pre and Post Test

Student	Pre-test responses on Confidence level	Post-test responses on Confidence level
Student A	3	4
Student B	3	4
Student C	1	3
Student D	2	3
Student E	2	4
Student F	4	4
Student G	3	2
Student H	1	2

*NB. Question 1 was omitted in the post-test as question 2 is more reflective of the confidence level. As such, Pre-test question 2 is compared with Post-test question 1.

Table 13. Summary of confidence level findings

Rating	Pre-test No of students	Post-test No of students
0	0	0
1	2	0
2	2	2
3	3	2
4	1	4

As the rating is on a scale of 1-5, the rating of 4 and 5 are considered as being confident. Thus, the results show that 1/8 – 12.5 % of the students felt confident solving problem sums at the pre - test level. However, at the Post test level 4/8 - 50% of the students felt confident. This is an increase of 37.5%.

Table 14. Strategies used when solving Mathematics problem sums

Strategy	No of student ticks	Strategy	No of student ticks
Working backwards	3	Assumption method	0
Listing	3	Before/ After	3
Draw a diagram	10	Unitary	9
Act it out	2	Simplify the problem	0
Guess and Check	3	Solve part of the problem	5
Look for a pattern	7		

Question: Which of these strategies do you use when solving Mathematics problem sums? Tick the strategy, Observations: The common heuristics used by the students are: Draw a diagram, Unitary, Look for a pattern and Solve part of the problem

Frequency of use of strategies

Question: How often do you apply these POLYA processes for solving problems? Circle one of the following on the scale:

I_________________I_________________I

I don't need to *Sometimes* *Always*

Table 15. Application of Polya Processes: Understand the Problem and Plan a Solution Strategy

Student	**Understand the Problem**		**Plan a Solution Strategy**	
	Pre-test	**Post test**	**Pre-test**	**Post test**
Student A	Sometimes	Always	Always	Always
Student B	Sometimes	Always	I don't need to	Always
Student C	Sometimes	Always	I don't need to	Always
Student D	Sometimes	I don't need to	Sometimes	Always
Student E	Sometimes	Always	Sometimes	Always
Student F	Always	Always	Always	Always
Student G	I don't need to	Always	Sometimes	Always
Student H	Always	Sometimes	I don't need to	Sometimes

Observations:

- 75% of the students find it necessary to understand the word problem before considering a suitable solution strategy.
- 87.5% of the students devise a strategy to solve the word problem.

Table 16. Application of Polya Processes: Solve the Problem and Check the solution

Student	Solve the Problem		Check the solution	
	Pre-test	Post test	Pre-test	Post test
Student A	Always	Always	Sometimes	Always
Student B	Sometimes	Always	Sometimes	Always
Student C	Always	Always	Sometimes	Sometimes
Student D	Sometimes	Always	Always	Always
Student E	Sometimes	Always	Sometimes	Sometimes
Student F	Always	Always	Sometimes	Sometimes
Student G	Always	Always	Always	I don't need to
Student H	Always	Sometimes	Sometimes	Sometimes

Observations:

- 87.5% of the students try their best to successfully solve the word problem.
- 37.5% of the students do their best to check their solutions. The remaining 62.5% do not consistently check their workings.

When these students were asked why they do not check their workings, they stated the following reasons:

In school, students do not need to use a checklist to check their workings and solutions. So they do not have the checking habit.

Students need to go on to other word problems and complete them quickly or else there will be some sums undone.

DISCUSSION

In this case study approach to evaluating the success of the PSUP programme at DAS, a series of significant results were found for the overall impact of the programme, and for components of the approach, which indicate that the PSUP is effective on many levels.

The post test scores for both the algebra and simultaneous concepts were greater between pre and post-tests and between review and post-tests. Thus, retention has taken place. The curriculum C-R-A, Try-Share-Learn-Apply and Polya's 4-step processes have definitely benefited the students in terms of their difficulties as students with dyslexia.

C-R-A

At the introductory stage, concrete resources were used for the students to visualize the concepts. This helped the students to be able to solve the word problems more readily. Students could also explore and use the resources to work on the other word problems should they need to act the situations presented in the word problems.

The Try-Share-Learn-Apply and Polya's steps

The Try stage enabled each student to work independently to solve the try problem. At the Share stage, each student had the opportunity to deep think and organise thinking processes to present the solution. The teacher and peers were thus able to listen, follow the presentation sequence and provide critical observations to help the student reflect and present the solution more effectively. Students were also able to pick up skills from each other. The Learn and Apply stages enabled the students to remember, retain and recall the learned solution/ strategy effectively.

Learn stage—Teaching Students to Read Word Problems Meaningfully

1. **Repeated Reading**
   - 1st reading: To get the main idea of the situations presented in the problem sums
   - 2nd reading: Read to make connections with the given information.
     - ◊ Subjects in the problem.
     - ◊ Their relationships.
     - ◊ What the question wants us to find

2. **Apply annotation**
   - Using symbols to highlight the important information.
     1. Use different colours to highlight different subjects
     2. Use different shapes to annotate different kinds of numbers
     3. Underline all mathematical terms
     4. Draw arrows to show the direction of comparison

The reading strategies helps students to organise their thinking and so document their mathematics equations and workings logically.

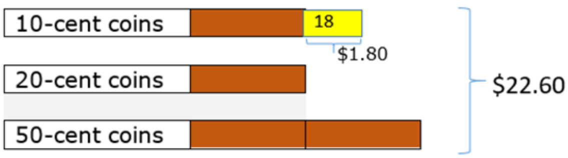
Good reading will lend itself into comprehension

2 units 1 unit

There were some 10-cent coins and twice as many 50-cent coins as 20-cent coins in a coin box. Asher added in some 10-cent coins into the box and the total amount became $22.60. There are 18 more 10-cent coins than 20-cent coins now.

How many more 10-cent coins than 50-cent coins are in the box now?

Representing the information to visualise and understand:



18 ten cents = $1.80

$22.60 - $1.80 = $20.80

Planning and Solving:

		Quantity
10 ¢	÷ 10 cents	1
20 ¢	÷ 10 cents	2
50 ¢	÷ 10 cents	5 (x2)
total		13

		Quantity
10 ¢	x 16	$1.60
20 ¢	x 16	$3.20
50 ¢	x 32	$16
total		$20.80

$20.80 ÷ 13 = 16

10-cent coins = 16 + 18 = 34

20-cent coins = 16

50-cent coins = 32

34 – 32 = 2

Answer: There are 2 more 10-cent coins than 50-cent coins

Figure 10. An example of a repeated single identity word problem—with annotation

3. Checking Written Solutions

As student worksheets have a checklist, Teachers can insist and check that when students submit their written work the relevant statements are ticked in the checklist at each lesson to ensure that checking becomes an automatic habit with the student. With regular practice it will not be cumbersome or cognitive overload for them.

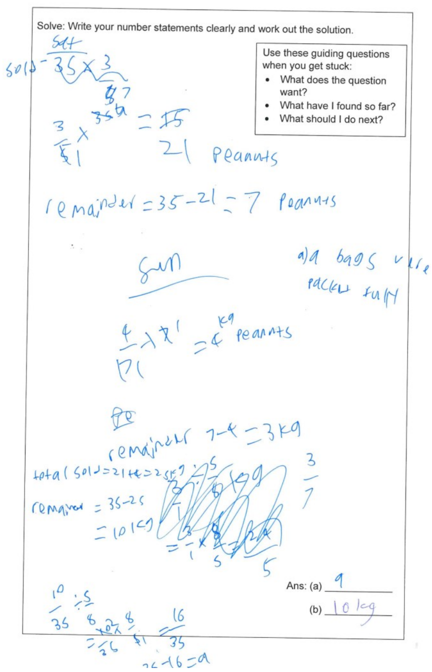
Table 17. Checklist in student's worksheet

PLEASE TICK (✓)	
	I have found what the question wants me to find
	My answers(s) make sense
	I have copied the numbers correctly from the question and my workings.
	My calculations are accurate
	I wrote the correct units on my answer

Observations how students wrote their workings before intervention and after intervention

A good point to note is that students wrote their workings more neatly and in logical sequence after intervention showing that they understood, planned and documented confidently.

Student working **before** intervention



Student working **after** intervention

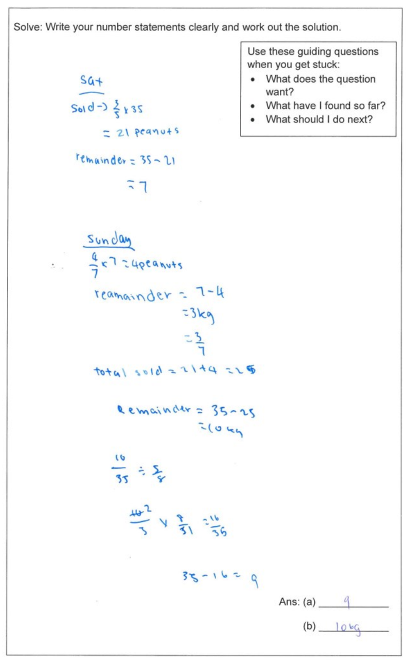


Figure 8. Sample of student workings

Discussion on students' feedback in the interview questions

At the Post test which was conducted in Week 10, students found it easier to answer questions on the algebra concept as compared to questions on the Simultaneous concept. The following methods were used by the students to understand the problem situations in the given problems:

- Apply Polya processes
- Re-read the word problem a number of times to fully understand the situations occurring in the word problem;
- underline keywords
- follow the sequence of events stated in the word problem scenario to have a clear picture about what is happening in the problem

The strategies used in solving the problems were:

- Work backwards from the last event to the first event; Draw a model; List and Act out the steps in sequence; Write mathematics statements - unitary method.

- When stuck in a challenging word problem, the students would read and reread the problem a few times to mentally visualise the scenarios in the question;

- draw pictures; use a strategy the child is comfortable in using and follow the sequence of events presented in the word problem

- For checking, some students put the answer into the question and worked out the solution. Others preferred to work backwards to see if they could arrive at the numbers given in the word problem.

Discussion on Student's Confidence at Pre and Post Intervention

As per the findings, at post intervention students are better able to apply Polya processes to understand questions, plan a solution strategy, solve and self correct. A repeated review of learned concepts has further improved students' ability to understand, plan and solve related word problems.

Conclusion—has the intervention benefited the students, how can the programme be improved / adjusted to better fulfil the needs of the students (teaching methods, resources)

In conclusion, the intervention has definitely benefited the students in the following ways - Firstly, they were able to complete the post test questions in a shorter time. b Secondly. they were more willing to approach and solve the questions with less apprehension as observed from their manner. Thirdly, they were able to solve the questions more accurately.

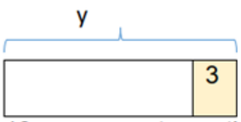
DIRECTIONS FOR FURTHER RESEARCH.

A. Improvements that could be put into place (instructional materials)

- More effort needs to be put in to create introductory activities (CRA in nature) and worksheets to tune in students to the algebra and simultaneous concepts. E.g. To better enable students to visualise situations for the Algebra concept, students could use omnifix cubes in DAS Maths resource / paper folding activity to act out the situations-

- In the question (Algebra concept)

There are y sweets in a packet. 3 sweets are taken out. How many sweets are left in the packet in terms of y?



Students could do a simple paper folding activity - a strip of paper to represent **y** could be folded at one end. The folded part represents the 3 sweets taken out. The number of sweets left in the packet is (**y - 3**) sweets.

For the second part of the question, unfold the 3 sweets part and add on another strip of paper perhaps of a different colour to visualise the situation.

There are (y - 3) + 10 sweets in the packet now.
Let y = 12. How many sweets are left in the packet now?
12 - 3 + 10 = 19. 10 sweets are left in the packet.

With more examples, students can visualise the situations in their mind and just write down the Maths equations and solve the sum.

- In the question (Simultaneous concept):

A pen and 2 pencil erasers cost $9.50. A pen and 4 pencil erasers cost $15.50. What is the cost of a pen?

pen	pencil erasers	amount
1	2	$9.50
1	4	$15.50

pen	eraser	amount
1	2	$9.50
1	4	$15.50

4 - 2 = 2
$15.50 - $9.50 = $6
2 pencil erasers cost $6

$9.50 - $6 = $3.50

A pen costs **$3.50**

- Students can use omnifix cubes - different colours to represent pen and pencil eraser to act out the situation presented in the word problem.

  For the representational stage, instead of writing mathematics equations and then eliminating the common multiple, students can insert the data into a table and solve the sum.

B. Improvements to Research Design

Improvements can be made to the research design of the case study as well. 8 students participated in the current study. Future studies could use a bigger sample size. Questionnaires could also be developed to ensure that students are able to easily understand and answer the questions. Questionnaires could also use options such as ticks or multiple choice instead of open ended questions so as to make it easier for students to answer the questions!

CONCLUSIONS

A case study approach to evaluating the PSUP approach to problem solving at DAS in Singapore has revealed the full potential of this system in addressing the known issues for dyslexic students in processing word problems. Further refinements to the system have been suggested to improve the impact in ongoing research.

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APPENDIX 1: Student questionnaire on their confidence level (Questions 1 to 3).

DYSLEXIA ASSOCIATION OF SINGAPORE

Problem Sums for Upper Primary PSUP

Student questionnaire

Name ____________________________ Date_______________

Answer the following questions as best as you can

1. On a scale of 1-5, How confident are you in solving Mathematics problem sums?
   Circle the number on the scale:

   |_______|_______|_______|_______|_______|

   Not confident 0 1 2 3 4 5 *Very confident*

2. Which of these strategies do you use when solving Mathematics problem sums?
   Tick the strategy

Working backwards		Assumption method	
Listing		Before/ After	
Draw a diagram		Unitary	
Act it out		Simplify the problem	
Guess and Check		Solve part of the problem	
Look for a pattern			

3. How often do you apply these POLYA processes for solving problems?
   Circle one of the following on the scale:

UNDERSTAND THE PROBLEM

|_________________|_________________|

I don't need to *Sometimes* *Always*

PLAN A SOLUTION STRATEGY

|_________________|_________________|

I don't need to *Sometimes* *Always*

SOLVE THE PROBLEM

|_________________|_________________|

I don't need to *Sometimes* *Always*

CHECK THE SOLUTION

|_________________|_________________|

I don't need to *Sometimes* *Always*

APPENDIX 2: Interview questionnaire on student thought processes in learning the algebra and simultaneous concepts.

INTERVIEW QUESTIONS

SCRIPT PRIOR TO INTERVIEW:

Dear student, thank you for being willing to take part in this interview aspect of this research project at the DAS. This study seeks to find out about the experiences of our students on the Problem Sums for Upper Primary programme and to measure the effectiveness of the programme.
Our interview today will last about 10-15 mins and I will ask you questions about the programme as well as about the test you have just completed. In class, you completed a consent form indicating that I have your permission (or not) to audio record our conversation.

Are you still ok with me recording (or not) our conversation today?

______ YES _______ NO

IF YES: Thank you! Please let me know if at any point you want me to turn off the recorder or keep something you said off the record.

IF NO: Thank you for letting me know. I will only take notes of our conversation.

Let's begin.

PART 1

1. On a scale of 1 to 5, how easy was this test for you?
   1 being very easy, 5 being very difficult.
2. Which question did you find the easiest?
3. Which question (s) did you find challenging?

PART 2

4. Let's walk through some of the questions. I would like you to describe to me in detail what went through your mind when you saw Question 1 (*Algebra question).*
5. What strategies did you use to help you understand this problem?
6. Was the strategy you used helpful? If not, what did you do?
7. Can you tell me more about the sequence of steps you took to solve this problem?
8. What happens after you had carried out your steps? What did you do to check your work?
9. How easy or difficult did you find this question?
   1 being very easy, 5 being very difficult
10. Let's move on to Question 2 *(Simultaneous Question).*
    Can you describe to me in detail how you approach this question?
11. What strategies did you use to help you understand this problem?
12. Was the strategy you used helpful? If not, what did you do?
13. Can you tell me more about the sequence of steps you took to solve this problem?
14. What happens after you have carried out your steps? What did you do to check your work?
15. How easy or difficult did you find this question?
    1 being very easy, 5 being very difficult.

PART 3

16. Let's look at one of the questions you found challenging. Could you tell me why this question was challenging for you?
17. Could you describe how you approached this question?
18. At which point did you get stuck? What did you do then?

PART 4

19. You have attended our Problem Sums class for 1 term. How do you find the lessons?
20. What are some strategies you have picked up from our lessons?
    (a) How likely will you use these strategies in your schoolwork?
    1 not at all, 5 I will definitely use them.
    (b) Tell me why you chose this score.
21. What do you like about the Problem Sums for Upper Primary (PSUP) lessons?
22. How can we improve our lessons for future batches of students?
23. Thank you for your participation. We have come to the end of the interview.
24. Do you have any questions to ask?

If not, thank you and you may take your leave.

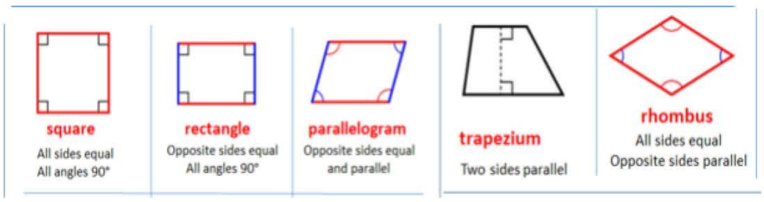
APPENDIX 3: Student Worksheet structure

PROBLEM SUMS FOR UPPER PRIMARY (P5)

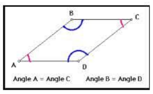
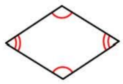
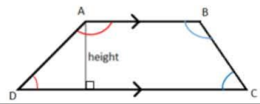
TOPIC/CONCEPT: QUADRILATERALS

Learning Outcomes

Students to be able to find the unknown angles in the Geometric figures using the properties of 4- sided **quadrilateral** figures:



* each angle of the square/rectangle is a right angle - 90°

Parallelogram	Rhombus	Trapezium
		
• Opposite sides are parallel. • Two pairs of parallel sides. • Opposite angles are equal. • Each pair of angles between the two parallel sides add up to 180 º	• All the sides are the same length. • Opposite sides are parallel. • Two pairs of parallel sides. • Opposite angles are equal. • Each pair of angles between the two parallel sides add up to 180 º.	• Opposite sides are parallel. • The pair of angles between the two parallel sides add up to 180 º Angle A + Angle D = 180 º Angle B + Angle C = 180 º

Instructions to students:

- Read the story in the sums meaningfully.
- Work through each sum systematically using the following approach:
  - Understand the problem
  - Devise a plan
  - Carry out the plan
- Check the solution
- Enjoy the sums!

Name: ______________________ Date : ______________

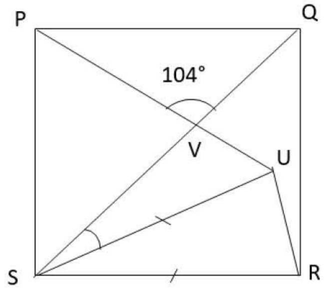
QUADRILATERALS

Name: ____________________________(P5) Date: _________________

Word Problem ①

In the figure, PQRS is a square.SR = SU, PVU and QVS are straight lines.

If ∠PVQ = 104°, find ∠VSU.



Understand the problem (*write the given essential information*)

1. ____

2. ____

3. ____

4. ____

What the question wants me to find:

1. ____

2. ____

When you have finished your work, check your solution using these questions:

Please tick (√)	
	Have I answered the question?
	Do my working steps and answer(s) make sense?
	Did I copy the numbers correctly from the question to my workings?
	Are my calculations accurate?
	Did I write the correct units in my answer?

2

Solve: Write your number statements clearly and work out the solution.

Use these guiding questions when you get stuck:

- What does the question want?
- What have I found so far?
- What should I do next?

Ans: ______________

3